

Twin Rotor Mimo System

ADVANCED TEACHING MANUAL 1

33-007-4M5



Feedback

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Notes



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PREFACE

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Preface

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1. INTRODUCTION

1.1. CONTENTS OF THIS MANUAL

This manual is the one of a set of 6 which comprise the documentation on the Twin Rotor MIMO System (TRMS).

The full set of documentation consist of:

- TRMS, Installation and Commissioning 33-007-0M5
- TRMS, Getting Started 33-007-1M5
- TRMS, Reference Guide 33-007-2M5
- TRMS, External Interface 33-007-3M5
- TRMS, Advanced Teaching Manual 1 33-007-4M5
- TRMS, Advanced Teaching Manual 2 33-007-5M5

The Advanced Teaching Manual 1 concentrates on the mathematical model and theoretical approaches to design using MATLAB and SIMULINK to model the TRMS dynamics.

It contains a detailed description of the TRMS mathematical model, identification procedures, PID-MIMO controller and a simple (1-Degree Of Freedom in the horizontal plane) LQ controller.

The TRMS mathematical model and identifications procedures allows a student to further develop an experiment using the EXTERNAL INTERFACE toolbox

The TRMS Advanced Teaching Manual 1 is delivered with software for use with MATLAB 5.1 and Simulink 2.1 or above.

The software provided includes:

- the Real Time Kernel for MATLAB 5,
- simulation, identification and control software,
- installation and testing procedures.



CHAPTER 1

Introduction

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1.2. MAIN CONTROL WINDOW

The *Main Control Window* is shown in Figure 1-1. It is invoked from the main MATLAB prompt by typing *hl*. It is described in detail in the Getting Started Manual – 33-007-1M5.

In addition to the menus and toolboxes described in the getting Started Manual (33-007-1M5), is the *Models & Control Systems* menu. This is opened by double-clicking the control (see Figure 1-2). From the *Models & Control Systems* menu the following programs are available:

In the columns:

- open-loop simulation models,
- PID simulation systems,
- LQ (linear quadratic) systems for 1 degree of freedom

In the rows:

- 1-DOF in vertical plane,
- 1-DOF in horizontal plane,
- 2-DOF (in both planes).

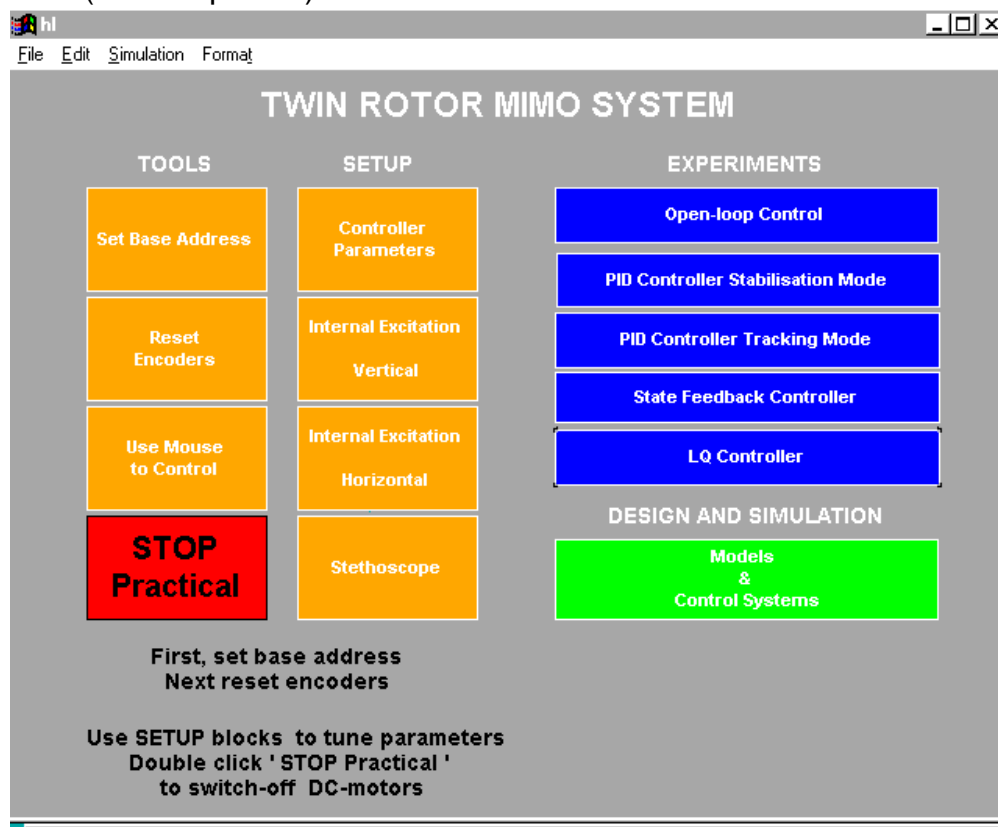


Figure 1-1: Main Control Window



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CHAPTER 1

Introduction

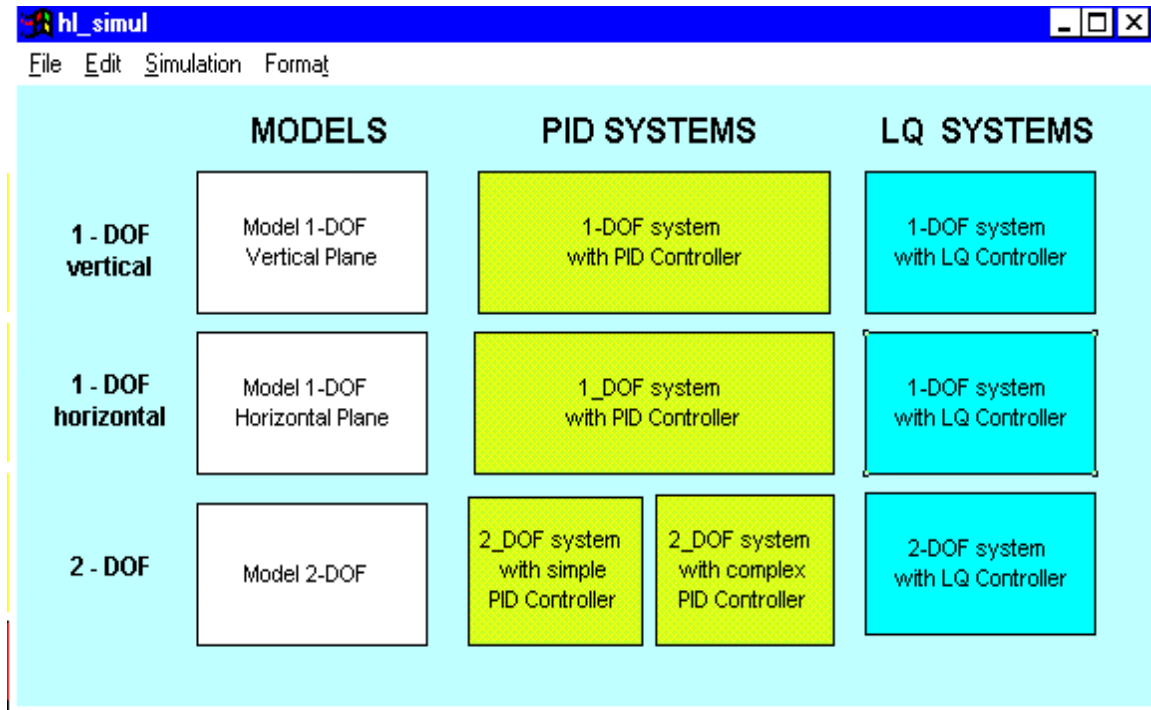


Figure 1-2: Models & Control Systems menu window.



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Introduction

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Notes



2. MATHEMATICAL MODEL

Figure 2-1 shows an aero-dynamical system similar to a helicopter. At both ends of a beam, pivoting on its base, there are two propellers driven by DC-motors. The articulated joint allows the beam to rotate in such a way that its ends move on spherical surfaces. There is a counter-weight fixed to the beam and it determines a stable equilibrium position. The system is balanced in such a way, that when the motors are switched off, the main rotor end of beam is lowered.

The controls of the system are the motors supply voltages.

The measured signals are: position of the beam in the space, that is, two position angles, and angular velocities of the rotors. Angular velocities of the beam are software reconstructed by differentiating and filtering measured position angles of the beam.

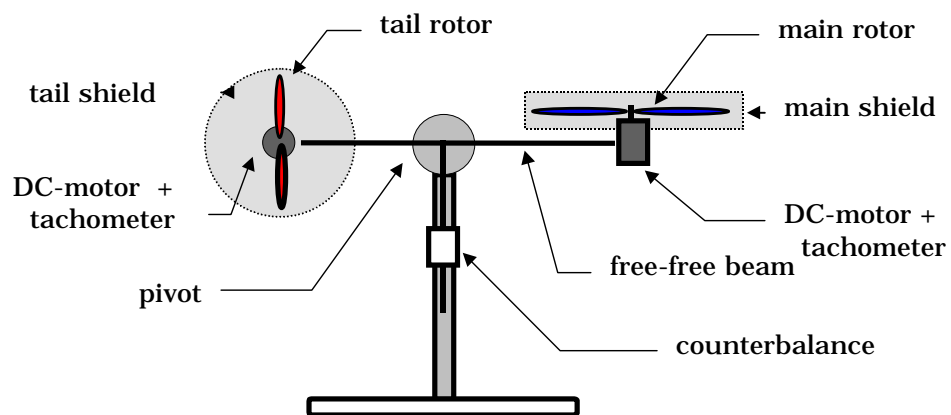


Figure 2-1: Aero-dynamical model of the Twin Rotor MIMO System

2.1. NON-LINEAR MODEL

The mathematical model is developed under some simplifying assumptions. First, it is assumed that the dynamics of the propeller subsystem can be described by first order differential equations. Further, it is assumed that friction in the system is of the viscous type. It is assumed also that the propeller-air subsystem could be described in accordance with the postulates of flow theory.

The above assumptions allow us to define the problem clearly. First, consider the rotation of the beam in the vertical plane i.e. around the **horizontal axis**. Having in mind that the driving torques are produced by the propellers, the rotation can be described in principle as the motion of a pendulum.



CHAPTER 2

Mathematical Model

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From Newton's second law of motion we obtain:

$$M_v = J_v \frac{d^2 \alpha_v}{dt^2} \quad (1)$$

where: M_v - is total moment of forces in the vertical plane,

J_v - is the sum of moments of inertia relative to the horizontal axis,

α_v - is the pitch angle of the beam.

Then:

$$M_v = \sum_{i=1}^4 M_{vi}, \quad J_v = \sum_{i=1}^8 J_{vi}$$

To determine the moments of gravity forces applied to the beam and making it rotate around the horizontal axis, consider the situation shown in Figure 2-2.

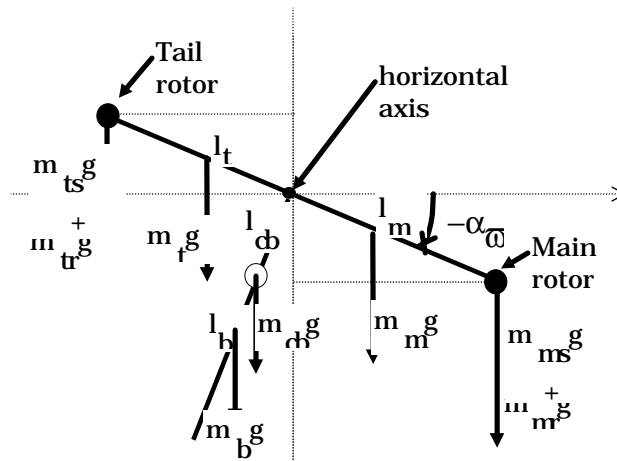


Figure 2-2: Gravity forces in TRMS corresponding to the return torque which determines the equilibrium position of the system.

$$M_{v1} = g \left\{ \left[\left(\frac{m_t}{2} + m_{tr} + m_{ts} \right) l_t - \left(\frac{m_m}{2} + m_{mr} + m_{ms} \right) l_m \right] \cos \alpha_v - \left(\frac{m_b}{2} l_b + m_{cb} l_{cb} \right) \sin \alpha_v \right\} \quad \text{Equation 1}$$

which may be expressed as

$$M_{v1} = g \{ [A - B] \cos \alpha_v - C \sin \alpha_v \} \quad \text{Equation 2}$$

where:

$$A = \left(\frac{m_t}{2} + m_{tr} + m_{ts} \right) l_t, \quad B = \left(\frac{m_m}{2} + m_{mr} + m_{ms} \right) l_m, \quad C = \left(\frac{m_b}{2} l_b + m_{cb} l_{cb} \right)$$



where:

| | | |
|----------|---|--|
| M_{v1} | - | is the return torque corresponding to the forces of gravity, |
| m_{mr} | - | is the mass of the main DC-motor with main rotor, |
| m_m | - | is the mass of main part of the beam, |
| m_{tr} | - | is the mass of the tail motor with tail rotor, |
| m_t | - | is the mass of the tail part of the beam, |
| m_{cb} | - | is the mass of the counter-weight, |
| m_b | - | is the mass of the counter-weight beam, |
| m_{ms} | - | is the mass of the main shield, |
| m_{ts} | - | is the mass of the tail shield, |
| l_m | - | is the length of main part of the beam, |
| l_t | - | is the length of tail part of the beam, |
| l_b | - | is the length of the counter-weight beam, |
| l_{cb} | - | is the distance between the counter-weight and the joint, |
| g | - | is gravitational acceleration. |

To determine the moments of propulsive forces applied to the beam consider the situation given in Figure 2-3.

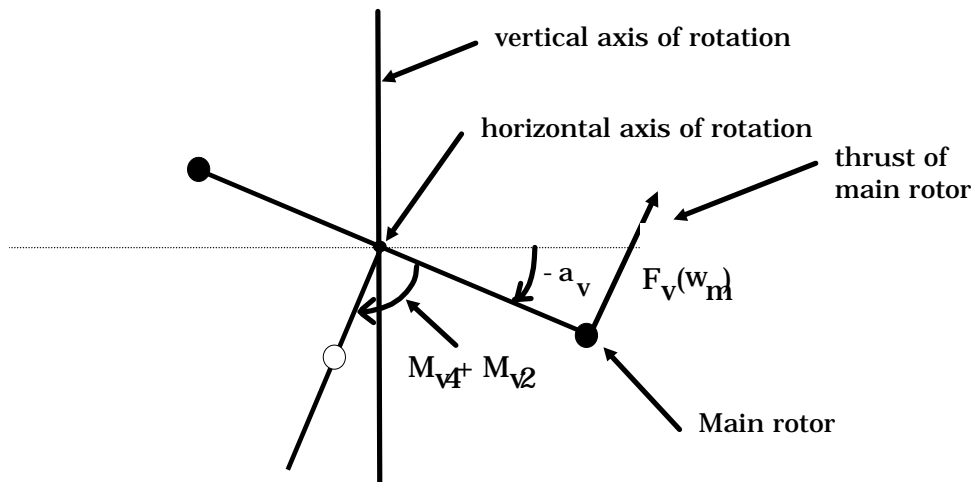


Figure 2-3: Propulsive force moment and friction moment in TRMS

$$M_{v2} = l_m F_v(\omega_m)$$

M_{v2} - is the moment of the propulsive force produced by the main rotor,

ω_m - is angular velocity of the main rotor,

$F_v(\omega_m)$ - denotes the dependence of the propulsive force on the angular velocity of the rotor. It should be measured experimentally (see section 3.2.1).



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$$M_{v3} = -\Omega_h^2 \left\{ \left(\frac{m_t}{2} + m_{tr} + m_{ts} \right) l_t + \left(\frac{m_m}{2} + m_{mr} + m_{ms} \right) l_m + \left(\frac{m_b}{2} l_b + m_{cb} l_{cb} \right) \right\} \sin \alpha_v \cos \alpha_v$$

or in the compact form:

$$M_{v3} = -\Omega_h^2 (A + B + C) \sin \alpha_v \cos \alpha_v$$

M_{v3} - is the moment of centrifugal forces corresponding to the motion of the beam around the vertical axis,

$$\text{and: } \Omega_h = \frac{d\alpha_h}{dt} \quad (2)$$

Ω_h - is the angular velocity of the beam around the vertical axis, α_h - is the azimuth angle of the beam.

$$M_{v4} = -\Omega_v k_v$$

M_{v4} - is the moment of friction depending on the angular velocity of the beam around the horizontal axis.

$$\text{where: } \Omega_v = \frac{d\alpha_v}{dt} \quad (3)$$

Ω_v - is the angular velocity around the horizontal axis,

k_v - is a constant.

According to Fig.2.3 we can determine components of the moment of inertia relative to the horizontal axis. Notice, that this moment is independent of the position of the beam.

$$J_{v1} = m_{mr} l_m^2,$$

$$J_{v2} = m_m \frac{l_m^2}{3},$$

$$J_{v3} = m_{cb} l_{cb}^2$$

$$J_{v4} = m_b \frac{l_b^2}{3},$$

$$J_{v5} = m_{tr} l_t^2,$$

$$J_{v6} = m_t \frac{l_t^2}{3}$$

$$J_{v7} = \frac{m_{ms}}{2} r_{ms}^2 + m_{ms} l_m^2,$$

$$J_{v8} = m_{ts} r_{ts}^2 + m_{ts} l_t^2$$



r_{ms} - is radius of main shield,
 r_{ts} - is radius of tail shield.

Similarly, we can describe the motion of the beam around the **vertical axis** having in mind that the driving torque's are produced by the rotors and that the moment of inertia depends on the pitch angle of the beam. The horizontal motion of the beam (around the vertical axis) can be described as a rotational motion of a solid mass.

$$M_h = J_h \frac{d^2 \alpha_h}{dt^2} \quad (4)$$

where: M_h - is the sum of moments of forces acting in the horizontal plane,
 J_h - is the sum of moments of inertia relative to the vertical axis.

Then: $M_h = \sum_{i=1}^2 M_{hi}$, $J_h = \sum_{i=1}^8 J_{hi}$.

To determine the moments of forces applied to the beam and making it rotate around the vertical axis consider the situation shown in Figure 2-4.

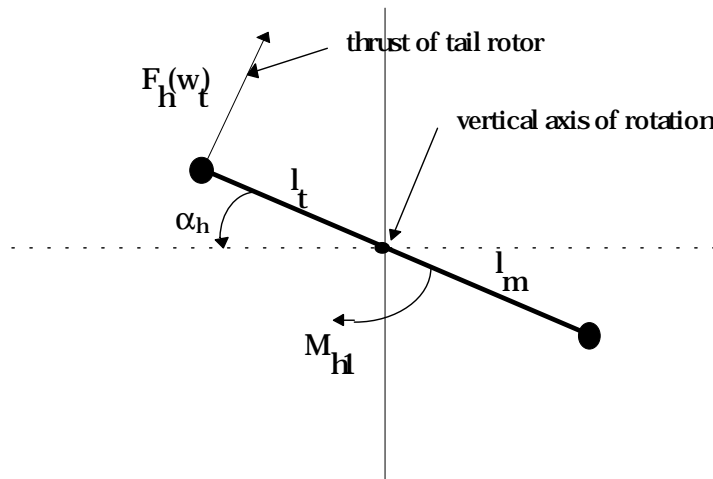


Figure 2-4: Moments of forces in horizontal plane (as seen from above)

$$M_{h1} = l_t F_h(\omega_t) \cos \alpha_v$$

ω_t - is the rotational velocity of tail rotor,

$F_h(\omega_t)$ - denotes the dependence of the propulsive force on the angular velocity of the tail rotor (should be determined experimentally, see section 3.2.2)



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$$M_{h2} = -\Omega_h k_h$$

M_{h2} - is the moment of friction depending on the angular velocity of the beam around the vertical axis,

k_h - is constant.

According to Fig.2.3 we can determine components of the moment of inertia relative to the vertical axis (it depends on pitch position of the beam).

$$J_{h1} = \frac{m_m}{3} (l_m \cos \alpha_v)^2,$$

$$J_{h2} = \frac{m_t}{3} (l_t \cos \alpha_v)^2,$$

$$J_{h3} = \frac{m_b}{3} (l_b \sin \alpha_v)^2$$

$$J_{h4} = m_{tr} (l_t \cos \alpha_v)^2,$$

$$J_{h5} = m_{mr} (l_m \cos \alpha_v)^2,$$

$$J_{h6} = m_{cb} (l_{cb} \sin \alpha_v)^2,$$

$$J_{h7} = \frac{m_{ts}}{2} r_{ts}^2 + m_{ts} (l_t \cos \alpha_v)^2,$$

$$J_{h8} = m_{ms} r_{ms}^2 + m_{ms} (l_m \cos \alpha_v)^2,$$

or in the compact form:

$$J_h = D \cos^2 \alpha_v + E \sin^2 \alpha_v + F,$$

where: D,E,F are constants:

$$D = \frac{m_b}{3} l_b^2 + m_{cb} l_{cb}^2,$$

$$E = \left(\frac{m_m}{3} + m_{mr} + m_{ms} \right) l_m^2 + \left(\frac{m_t}{3} + m_{tr} + m_{ts} \right) l_t^2,$$

$$F = m_{ms} r_{ms}^2 + \frac{m_{ts}}{2} r_{ts}^2$$

Using (1)-(4) we can write the equations describing the motion of the system as follows:

$$\frac{dS_v}{dt} = \frac{l_m F_v(\omega_m) - \Omega_v k_v + g((A - B) \cos \alpha_v - C \sin \alpha_v) - \frac{1}{2} \Omega_h^2 (A + B + C) \sin 2\alpha_v}{J_v} \quad (5)$$



$$\frac{d\alpha_v}{dt} = \Omega_v, \quad \Omega_v = S_v + \frac{J_{tr} \omega_t}{J_v} \quad (6)$$

$$\frac{dS_h}{dt} = \frac{I_t F_h(\omega_t) \cos \alpha_v - \Omega_h k_h}{J_h} = \frac{I_t F_h(\omega_t) \cos \alpha_v - \Omega_h k_h}{D \sin^2 \alpha_v + E \cos^2 \alpha_v + F} \quad (7)$$

$$\frac{d\alpha_h}{dt} = \Omega_h, \quad \Omega_h = S_h + \frac{J_{mr} \omega_m \cos \alpha_v}{J_h} = S_h + \frac{J_{mr} \omega_m \cos \alpha_v}{D \sin^2 \alpha_v + E \cos^2 \alpha_v + F} \quad (8)$$

where: J_{tr} - moment of inertia in DC-motor -tail propeller subsystem,
 J_{mr} - moment of inertia in DC-motor -main propeller subsystem,
 S_v - angular momentum in vertical plane for the beam,
 S_h - angular momentum in horizontal plane for the beam.

Additional terms in equations (6) and (8) follows from principle of conservation of angular momentum.

It should be noted that angular velocities are non-linear functions of the input voltage of the DC-motor. Thus we have two additional equations:

$$\frac{du_{vv}}{dt} = \frac{1}{T_{mr}} (-u_{vv} + u_v), \quad \omega_m = P_v(u_{vv}) \quad (9)$$

$$\frac{du_{hh}}{dt} = \frac{1}{T_{tr}} (-u_{hh} + u_h), \quad \omega_t = P_h(u_{hh}) \quad (10)$$

where: T_{mr} - time constant of main motor- propeller system,
 T_{tr} - time constant of tail motor- propeller system .

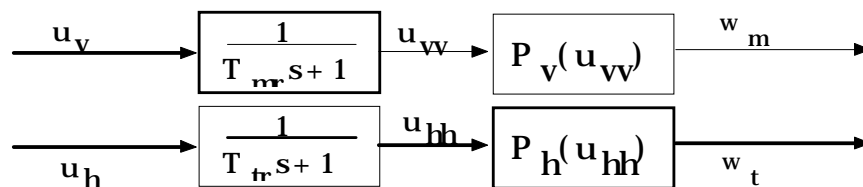


Figure 2-5: Block diagram of equation (9) and (10)



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The above model of the motor-propeller dynamics is obtained by substituting the non-linear system by a serial connection of a linear dynamic system and static non-linearity (Figure 2-5).

2.2. STATE EQUATIONS

Finally, the mathematical model of the TRMS becomes a set of four non-linear differential equations with two linear differential equations and four non-linear functions. Variables of the model (5) - (10) can be classified as follows (Figure 2-6):

$$U = \begin{bmatrix} u_h \\ u_v \end{bmatrix} \text{ is the input, } X = \begin{bmatrix} S_h \\ \alpha_h \\ u_{hh} \\ S_v \\ \alpha_v \\ u_{vv} \end{bmatrix} \text{ is the state, and } Y = \begin{bmatrix} \Omega_h \\ \alpha_h \\ \omega_t \\ \Omega_v \\ \alpha_v \\ \omega_m \end{bmatrix} \text{ is the output vector.}$$

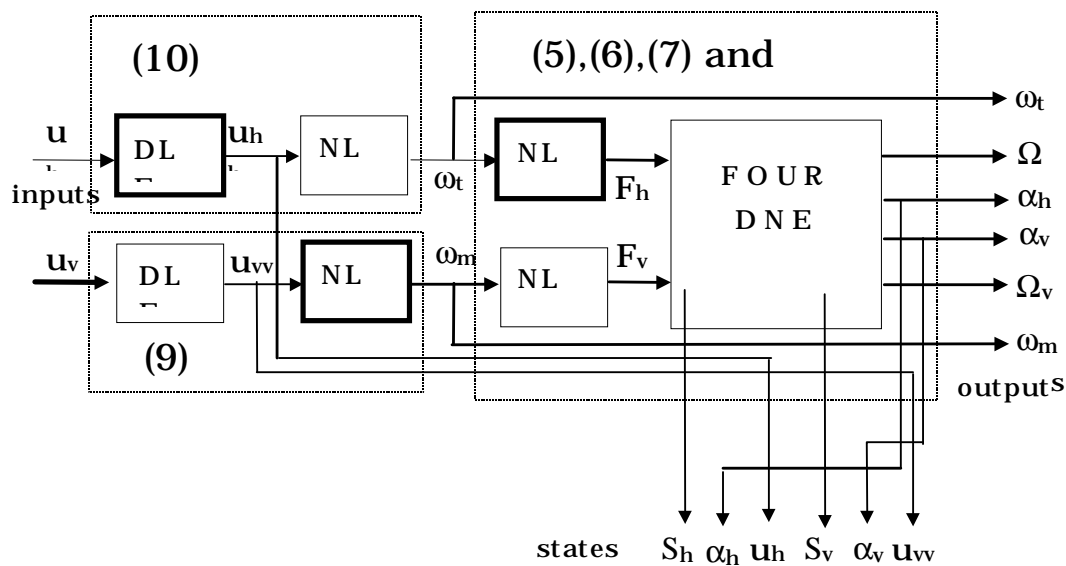


Figure 2-6: Block diagram of the model equations (DLE denotes a linear differential equation, NL denotes non-linearity, and DNE denotes a non-linear differential equation)

In order to apply the model (5) - (10) for control of TRMS the parameters and non-linear functions should be determined first. They can be divided in to three groups:

- physical parameters,
- non-linear static characteristics,
- time constants of the linear part of the model.



3. BASIC PARAMETERS AND CHARACTERISTICS

3.1. PHYSICAL PARAMETERS

To obtain the values of the model parameters it is necessary to make some measurements. First, geometrical dimensions and moving masses of TRMS should be measured. Following are the results of measurements for a given laboratory set-up. The notations are explained in Figure 2-2, Figure 2-3 and Figure 2-4.

| | | |
|------------------|------------------|------|
| | $m_{tr} = 0.206$ | [kg] |
| $l_t = 0.25$ | $m_{mr} = 0.228$ | [kg] |
| $l_m = 0.24$ | $m_{cb} = 0.068$ | [kg] |
| $l_b = 0.26$ | $m_t = 0.0155$ | [kg] |
| $l_{cb} = 0.13$ | $m_m = 0.0145$ | [kg] |
| $r_{ms} = 0.155$ | $m_b = 0.022$ | [kg] |
| $r_{ts} = 0.10$ | $m_{ts} = 0.165$ | [kg] |
| | $m_{ms} = 0.225$ | [kg] |

Using the above measurements the moment of inertia about the horizontal axis can be calculated as:

$$J_v = \sum_i^8 J_{iv} = 0.055846 \text{ [kg m}^2\text{]}$$

The terms of the sum are calculated from elementary physics laws:

$$\begin{aligned} J_{v1} &= m_{tr} l_t^2 = 0.012875 \text{ [kg m}^2\text{]} \\ J_{v2} &= m_{cb} l_{cb}^2 = 0.001149 \text{ [kg m}^2\text{]} \\ J_{v3} &= m_{mr} l_m^2 = 0.013132 \text{ [kg m}^2\text{]} \\ J_{v4} &= m_t l_t^2 / 3 = 0.000322 \text{ [kg m}^2\text{]} \\ J_{v5} &= m_m l_m^2 / 3 = 0.000278 \text{ [kg m}^2\text{]} \\ J_{v6} &= m_b l_b^2 / 3 = 0.000495 \text{ [kg m}^2\text{]} \\ J_{v7} &= m_{ms} (r_{ms}^2 / 2 + l_m^2) = 0.015622 \text{ [kg m}^2\text{]} \\ J_{v8} &= m_{ts} (r_{ts}^2 + l_t^2) = 0.011962 \text{ [kg m}^2\text{]} \end{aligned}$$



CHAPTER 3

Basic Parameters And Characteristics

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The calculated moment of inertia about the vertical axis is:

$$J_h = \sum_i^8 J_{hi} ,$$

where the terms of the sum are:

$$J_{h2} = m_t (l_t \cos \alpha_v)^2 / 3 = 0.0003229 \cos^2 \alpha_v \quad [\text{kg m}^2]$$

$$J_{h1} = m_m (l_m \cos \alpha_v)^2 / 3 = 0.0002784 \cos^2 \alpha_v \quad [\text{kg m}^2]$$

$$J_{h3} = m_b (l_b \sin \alpha_v)^2 / 3 = 0.0004595 \sin^2 \alpha_v \quad [\text{kg m}^2]$$

$$J_{h5} = m_{mr} (l_m \cos \alpha_v)^2 = 0.013132 \cos^2 \alpha_v \quad [\text{kg m}^2]$$

$$J_{h4} = m_{tr} (l_t \cos \alpha_v)^2 = 0.012875 \cos^2 \alpha_v \quad [\text{kg m}^2]$$

$$J_{h6} = m_{cb} (l_{cb} \sin \alpha_v)^2 = 0.0011492 \sin^2 \alpha_v \quad [\text{kg m}^2]$$

$$J_{h7} = m_{ts} (r_{ts}^2 / 2 + l_t^2 \cos^2 \alpha_v) = 0.000825 + 0.010312 \cos^2 \alpha_v \quad [\text{kg m}^2]$$

$$J_{h8} = m_{ms} (r_{ms}^2 + l_m^2 \cos^2 \alpha_v) = 0.00540 + 0.0129611 \cos^2 \alpha_v \quad [\text{kg m}^2]$$

giving finally (Figure 3-1):

$$J_h = \sum_i^8 J_{hi} = D \cos^2 \alpha_v + E \sin^2 \alpha_v + F = 0.04901 \cos^2 \alpha_v + 0.0016065 \sin^2 \alpha_v + 0.0063306$$

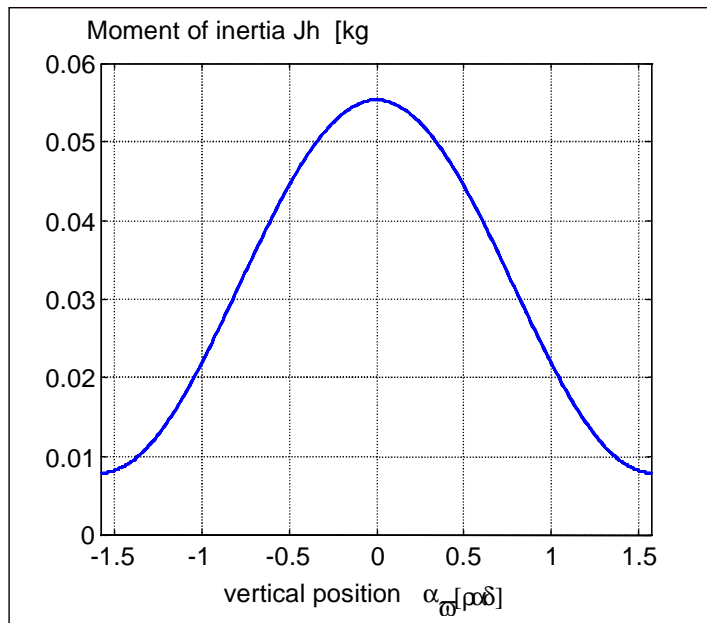


Figure 3-1: Moment of inertia J_h



The returning torque from gravity forces is given by (see Equation 1, page 2-2)

$$M_{v1} = g \left\{ \left[\left(\frac{m_t}{2} + m_{tr} + m_{ts} \right) l_t - \left(\frac{m_m}{2} + m_{mr} + m_{ms} \right) l_m \right] \cos \alpha_v - \left(\frac{m_b}{2} l_b + m_{cb} l_{cb} \right) \sin \alpha_v \right\}$$

1 2 3
4 5 6
7 8

The measured values of the eight terms in the equation above are shown below:

$$\begin{aligned}
 4 &= -9.81 m_m l_m \cos \alpha_v / 2 = -0.001711 \cos \alpha_v \quad [\text{N m}] \\
 5 &= -9.81 m_{mr} l_m \cos \alpha_v = -0.053808 \cos \alpha_v \quad [\text{N m}] \\
 6 &= -9.81 m_{ms} l_m \cos \alpha_v = -0.0531 \cos \alpha_v \quad [\text{N m}] \\
 1 &= +9.81 m_t l_t \cos \alpha_v / 2 = 0.0019375 \cos \alpha_v \quad [\text{N m}] \\
 2 &= +9.81 m_{tr} l_t \cos \alpha_v = 0.0515 \cos \alpha_v \quad [\text{N m}] \\
 3 &= +9.81 m_{ts} l_t \cos \alpha_v = 0.04125 \cos \alpha_v \quad [\text{N m}] \\
 7 &= -9.81 m_b l_b \sin \alpha_v / 2 = 0.00286 \sin \alpha_v \quad [\text{N m}] \\
 8 &= -9.81 m_{cb} l_{cb} \sin \alpha_v = 0.00884 \sin \alpha_v \quad [\text{N m}]
 \end{aligned}$$

giving (Figure 3-2):

$$M_{v1} = 9.81 (-0.0139315 \cos \alpha_v + 0.0117016065 \sin \alpha_v) \quad [\text{N m}].$$

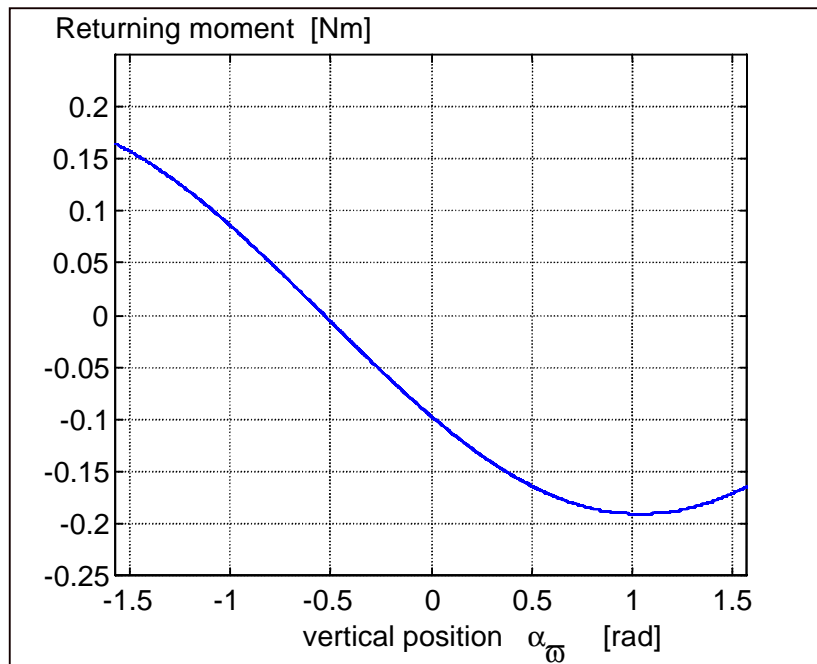


Figure 3-2: Returning moment of gravity forces M_{v1}



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The moment of centrifugal forces is:

$$M_{v3} = \sum_i^6 M_{v3,i} ,$$

where:

$$M_{v3,1} = (m_{tr} + m_{ts})l_t^2 \Omega_h^2 \cos\alpha_v \sin\alpha_v = 0.0231875 \Omega_h^2 \cos\alpha_v \sin\alpha_v \text{ [N m]}$$

$$M_{v3,2} = m_t l_t^2 \Omega_h^2 \cos\alpha_v \sin\alpha_v / 2 = 0.0002421 \Omega_h^2 \cos\alpha_v \sin\alpha_v \text{ [N m]}$$

$$M_{v3,3} = m_b l_b^2 \Omega_h^2 \cos\alpha_v \sin\alpha_v / 2 = 0.0003718 \Omega_h^2 \cos\alpha_v \sin\alpha_v \text{ [N m]}$$

$$M_{v3,4} = m_{cb} l_{cb}^2 \Omega_h^2 \cos\alpha_v \sin\alpha_v = 0.0011492 \Omega_h^2 \cos\alpha_v \sin\alpha_v \text{ [N m]}$$

$$M_{v3,5} = m_m l_m^2 \Omega_h^2 \cos\alpha_v \sin\alpha_v / 2 = 0.0002018 \Omega_h^2 \cos\alpha_v \sin\alpha_v \text{ [N m]}$$

$$M_{v3,6} = (m_{mr} + m_{ms})l_m^2 \Omega_h^2 \cos\alpha_v \sin\alpha_v = 0.02523028 \Omega_h^2 \cos\alpha_v \sin\alpha_v \text{ [N m]}$$

giving finally (Figure 3-3):

$$M_{cf} = \sum_i^6 M_{icf} = 0.05038268 \Omega_h^2 \cos\alpha_v \sin\alpha_v \text{ [N m]} .$$

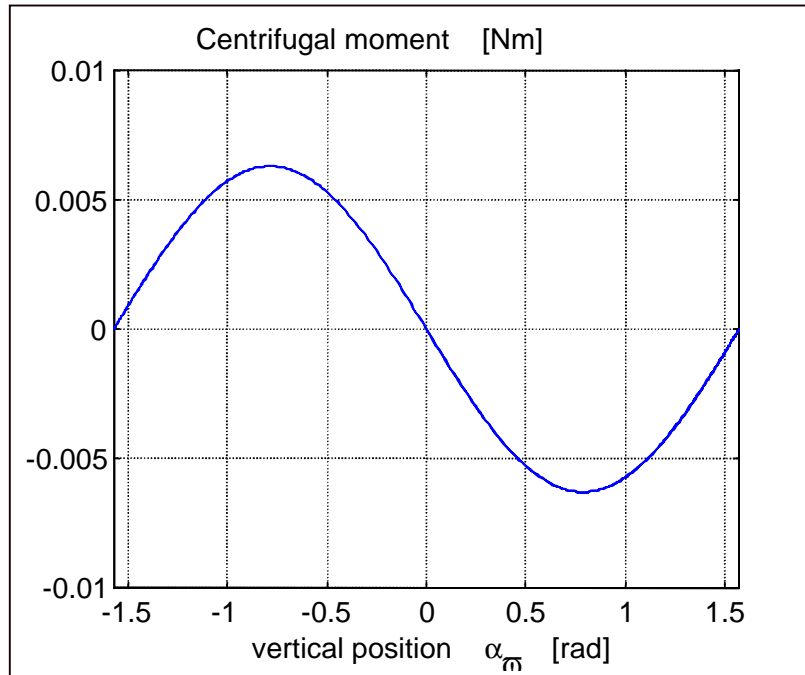


Figure 3-3: Centrifugal moment for $\Omega_h=1$ [rad/s]



3.2. STATIC CHARACTERISTICS

It is necessary to identify the following non-linear functions:

- Two non-linear input characteristics determining dependence of DC- motor rotational speed on input voltage:

$$\omega_m = P_v(u_{vv}), \quad \omega_t = P_h(u_{hh})$$

- Two non-linear characteristics determining dependence of propeller thrust on DC- motor rotational speeds:

$$F_h = F_h(\omega_t), \quad F_v = F_v(\omega_m)$$

The time constants T_{mr} and T_{tr} (see equations (9) and (10)) should be determined experimentally.

The static characteristics of the propellers should be measured in the case when not delivered with equipment documentation or if the propellers were changed by user. In this case a proper electronic balance with voltage output [0-10V] is needed. The recommended range of input force is from 0 to 2 N. The measurement procedures are described in the following sections.

3.2.1. Main rotor characteristics

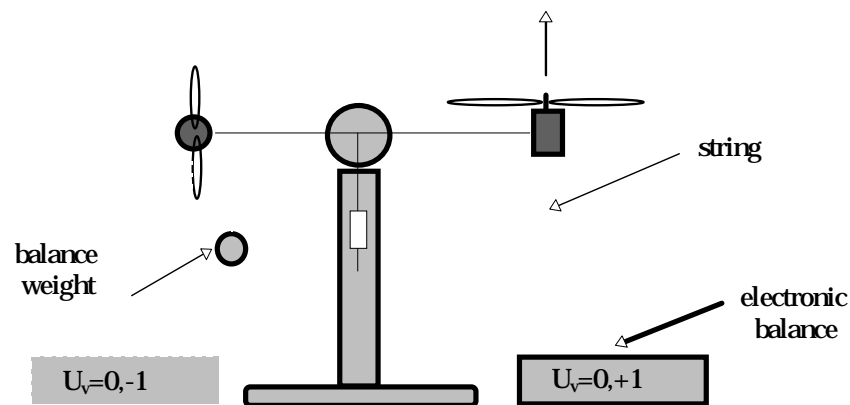


Figure 3-4: Measuring of main rotor characteristics.

Note: The experiments with the electronic balance described in this and the next sections are not recommended for students. They show only methods of measuring the static characteristics of the DC-Motor and propeller system.

To make the measurements correctly, first block the beam so that it cannot rotate around the vertical axis. Place the electronic balance under the beam in such a way that it is pulled vertically up by the propeller. To balance the beam in the horizontal position attach a weight to the beam (as in Figure 3-4). Connect the voltage output of the electronic balance to A/D input No. 3 of the PCL-812PG or RT-DAC data acquisition board.



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To measure the characteristic execute the following command sequence:

```
pw=fhl_call('getpwhoriz');
pw(1:2)=[ 0 0 ];
dummy=fhl_call('setpwvert',pw);
dummy=fhl_call('setpwhoriz',pw);
a=fhl_call('setalgn',1);
disp('POWER ON');
pause
arr = [];
vel = [];
ctrl = [];
count = 1;
for i = 0 : -0.25 : -10           % line to be changed
    pw(1:2)=[ 0 0.1*i ];
    dummy=fhl_call('setpwhoriz',pw);
    h=fhl_call('gethistory');
    pause(5)
    h=fhl_call('gethistory');
    ctrl(count) = i;
    arr(count) = mean( h(12,:) );
    vel(count) = mean( h(7,:) );
    count = count + 1;
    plot( [vel;arr]' );
end
pw(1:2)=[ 0 0 ];
dummy=fhl_call('setpwvert',pw);
dummy=fhl_call('setpwhoriz',pw);
fhl_call('unloadlibrary')
```

The above routine sends a normalised input signal (from **0** to **-1**) which corresponds the 0 to-5[V] on the output of PCL-812 board to the main DC-motor and reads main rotor rotational speed and thrust. To measure the characteristics for input range from **0** to **+1** change the balance position as in Fig.3.4 and execute the routine having previously changed the 12th line to:

```
for i = 0 : 0.25 : +10
```

The obtained measurements should be translated from internal system units to physical units (newtons [N]), taking into account the construction of the balance. Example characteristics are given in Figure 3-5.

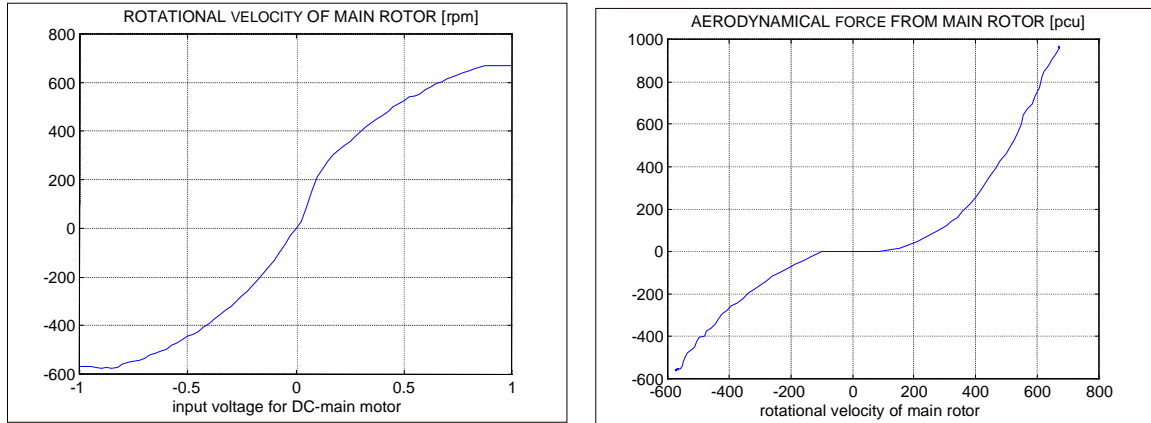


Figure 3-5: Measured characteristics of the main rotor.

For further applications the measured characteristics should be substituted by their polynomial approximations. For this purposes one can use the MATLAB ***polyfit.m*** function. An example is given in Figure 3-6. The polynomials:

$$\omega_m(u_{vv}) = 90.99u_{vv}^6 + 599.73u_{vv}^5 - 129.26u_{vv}^4 - 1238.64u_{vv}^3 + 63.45u_{vv}^2 + 1283.41u_{vv}$$

$$F_v(\omega_m) = -3.48 \cdot 10^{-12} \omega_m^5 + 1.09 \cdot 10^{-9} \omega_m^4 + 4.123 \cdot 10^{-6} \omega_m^3 - 1.632 \cdot 10^{-4} \omega_m^2 + 9.544 \cdot 10^{-2} \omega_m$$

were used in this case.

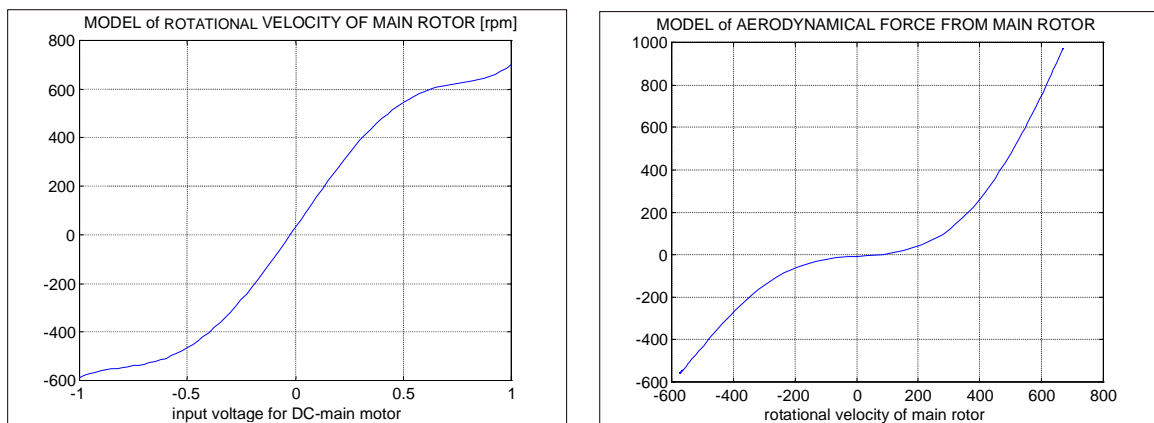


Figure 3-6: Polynomial approximation of the main rotor characteristics .



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3.2.2. Tail rotor characteristics

Figure 3-7 shows the laboratory set-up for measuring the thrust and rotational speed of the tail rotor. **Notice that the beam was turned by 90° (after loosening the screw) so that the tail rotor axis is vertical now !!.**

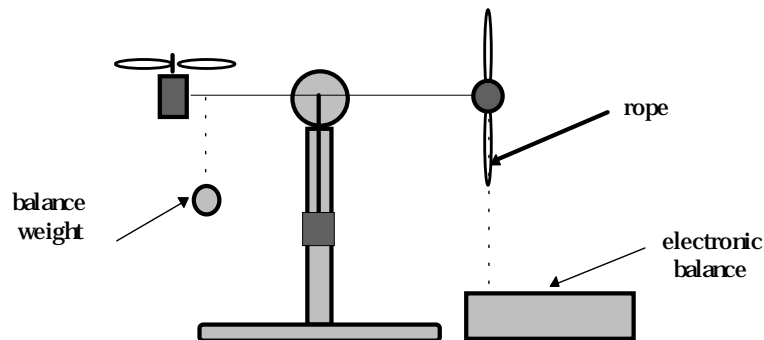


Figure 3-7: Laboratory set-up for tail rotor characteristics

To measure the characteristic execute the following routine:

```
pw=fhl_call('getpwhoriz');
pw(1:2)=[ 0 0 ];
dummy=fhl_call('setpwvert',pw);
dummy=fhl_call('setpwhoriz',pw);
a=fhl_call('setalgn',1);
disp('POWER ON');
pause
arr = [];
vel = [];
ctrl = [];
count = 1;
for i = 0 : -0.25 : -10           % line to be changed
    pw(1:2)=[ 0 0.1*i ];
    dummy=fhl_call('setpwvert',pw);
    h=fhl_call('gethistory');
    pause(5)
    h=fhl_call('gethistory');
    ctrl( count ) = i;
    arr(count) = mean( h(12,:) );
    vel(count) = mean( h(6,:) );
    count = count + 1;
    plot( [vel;arr]' );
end
pw(1:2)=[ 0 0 ];
dummy=fhl_call('setpwvert',pw);
dummy=fhl_call('setpwhoriz',pw);
fhl_call('unloadlibrary')
```



The above routine sends a normalised input signal (from **0** to **-1**) which corresponds the 0 to-5[V] on the output of PCL-812 card to the tail DC-motor and reads tail rotor rotational speed and thrust. Than to measure the characteristics for input range from **0** to **+1** change the balance position as in Fig.3.7 and execute the routine having previously changed the 12th line to:

for i = 0 : 0.25 : +10

The obtained measurements should be translated from internal system units to physical units (newtons [N]) taking into account the construction of the balance.
 Example characteristics are given in Figure 3-8.

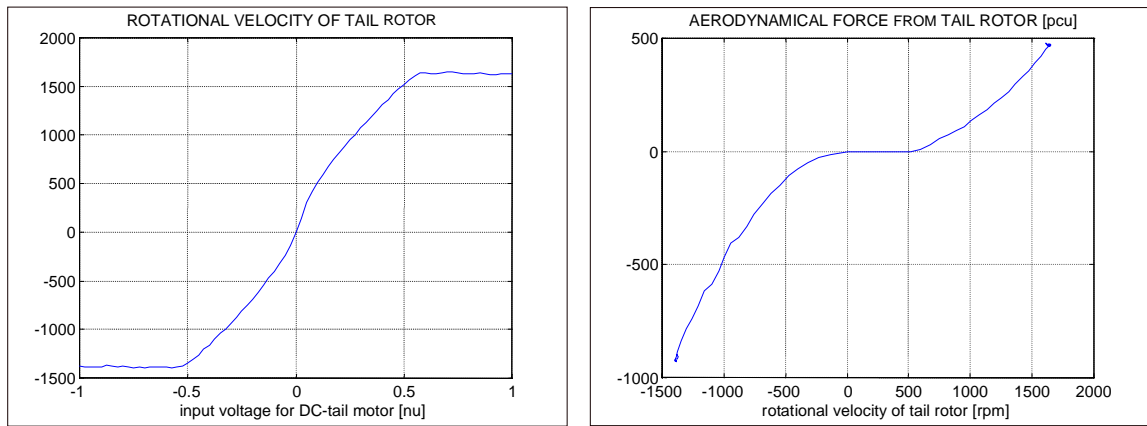


Figure 3-8: Example characteristics measured for the tail rotor.

For further applications the measured characteristics should be substituted by their polynomial approximations. An example is given in Fig.3.9. The polynomials:

$$\omega_t(u_{hh}) = 2020u_{hh}^5 - 194.69u_{hh}^4 - 4283.15u_{hh}^3 + 262.27u_{hh}^2 + 3796.83u_{hh}$$

$$F_h(\omega_t) = -3 \cdot 10^{-14} \omega_t^5 - 1.595 \cdot 10^{-11} \omega_t^4 + 2.511 \cdot 10^{-7} \omega_t^3 - 1.808 \cdot 10^{-4} \omega_t^2 + .0801 \omega_t$$

were used in this case.



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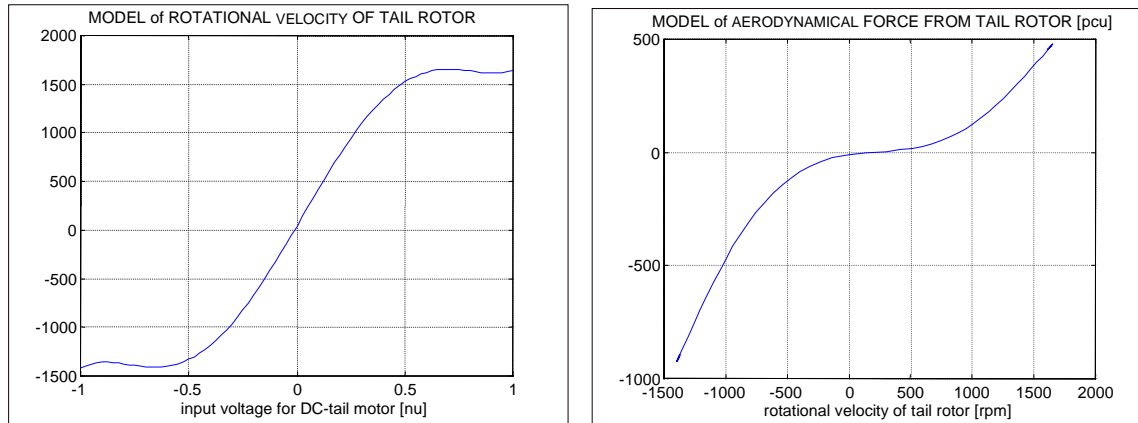


Figure 3-9: Polynomial approximation of tail rotor characteristics



4. MODELS OF THE TRMS

This section presents Simulink models of TRMS. The models are based on non-linear equations given in Section 2.1.

Three models are considered :

- 1-DOF (1-Degree Of Freedom) model of the horizontal subsystem of TRMS,
- 1-DOF model of the vertical subsystem of TRMS,
- 2-DOF complete model of TRMS.

The 1-DOF models describe the situation when motion in one of the axis is blocked.

Creation of two 1-DOF separate models for horizontal and vertical plane was necessary because there is no natural way to split 2-DOF complex model into two independent parts.

Both horizontal and vertical models are used for identification of 1-DOF systems. The third model can be used for sensitivity testing of the open system for changes of previously identified coefficients. In general, these models help to understand physical laws which govern TRMS and can be used for closed-loop control systems design (as described in the following sections).

To open these models type in the MATLAB *Command Window* respectively:

- ***m_1dofh.mdl*** - for the horizontal plane model,
(convention used for the function name: *m_1dofh* = model 1-DOF horizontal),
- ***m_1dofv.mdl*** - for the vertical plane model,
- ***m_2dof.mdl*** - for the complex model.

You can open these models from the *MODELS* column in the *Models & Control Systems* window (Figure 1-2).

4.1. MODEL OF THE HORIZONTAL PART OF TRMS

Figure 4-1 shows the Simulink 1-DOF model of the horizontal part of TRMS. Figure 4-1a shows the grouped model with scopes for the visualisation of input, position and velocity. It can be used to observe the behaviour of the open loop system. You can change the shape and level of the input signal by selecting the type of input block from the *Simulink Library* e.g. *Pulse Generator* , *Signal Generator* block or other. Figure 4-1b shows the contents of the grouped model block. Figure 4-1c shows details of the *driving torque* block and the *speed of tail rotor* block. Figure 4-1d shows the structure of the *rotational speed of main rotor* block and the *aero-force* block. Here you can examine the static characteristic functions: $\omega_t = P_h(u_{hh})$ and $F_h = F_h(\omega_t)$, and modify them if necessary.



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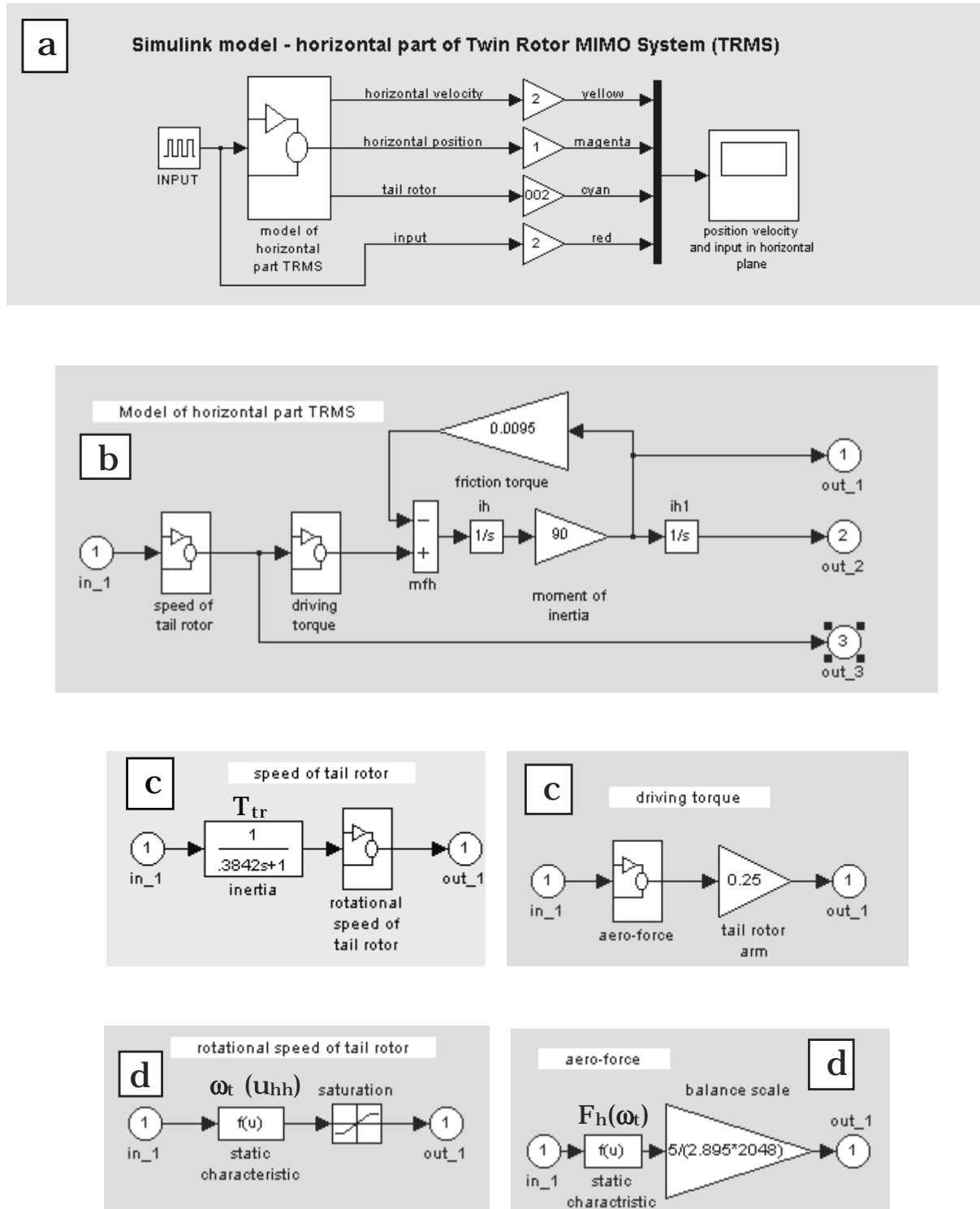


Figure 4-1: Model of horizontal part of TRMS general form; b), c) and d) show details of the blocks



4.2. MODEL OF THE VERTICAL PART OF TRMS

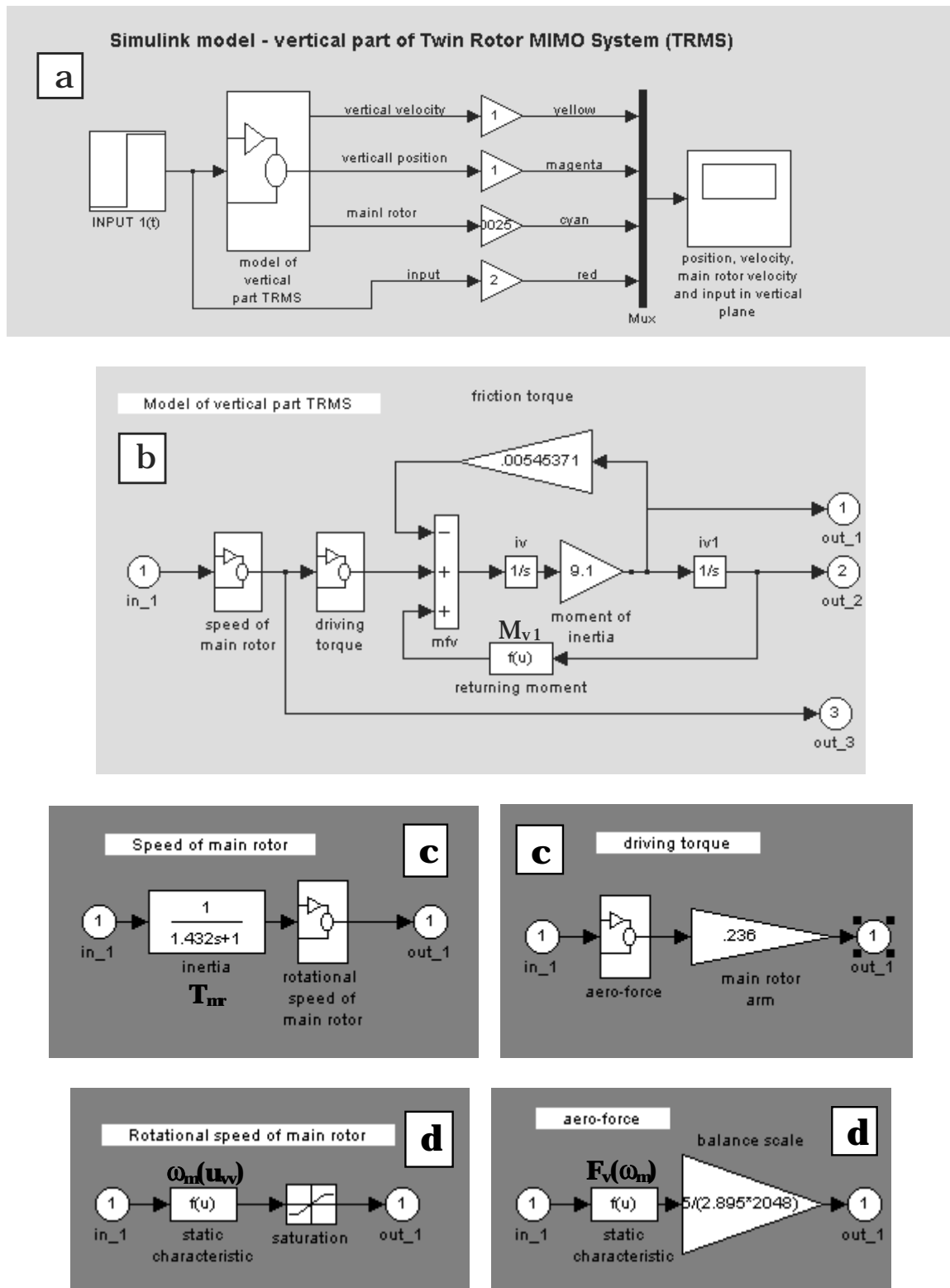


Figure 4-2: Model of vertical part of TRMS – a) general form; b), c), and d) show details of the grouped blocks



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Figure 4-2 shows 1-DOF model of the vertical part of TRMS. Figures Figure 4-2b, Figure 4-2c and Figure 4-2d give details of the grouped blocks.

4.3. 2- DOF MODEL OF TRMS

The 2-DOF model of TRMS is shown in Figure 4-3. The model is based on the diagram from Figure 2-4 and equations given in Section 2. In this form the model can be used for observation of all the state variables in the open loop mode and for identification. The model can also be used for developing closed-loop control systems as described in the following sections.

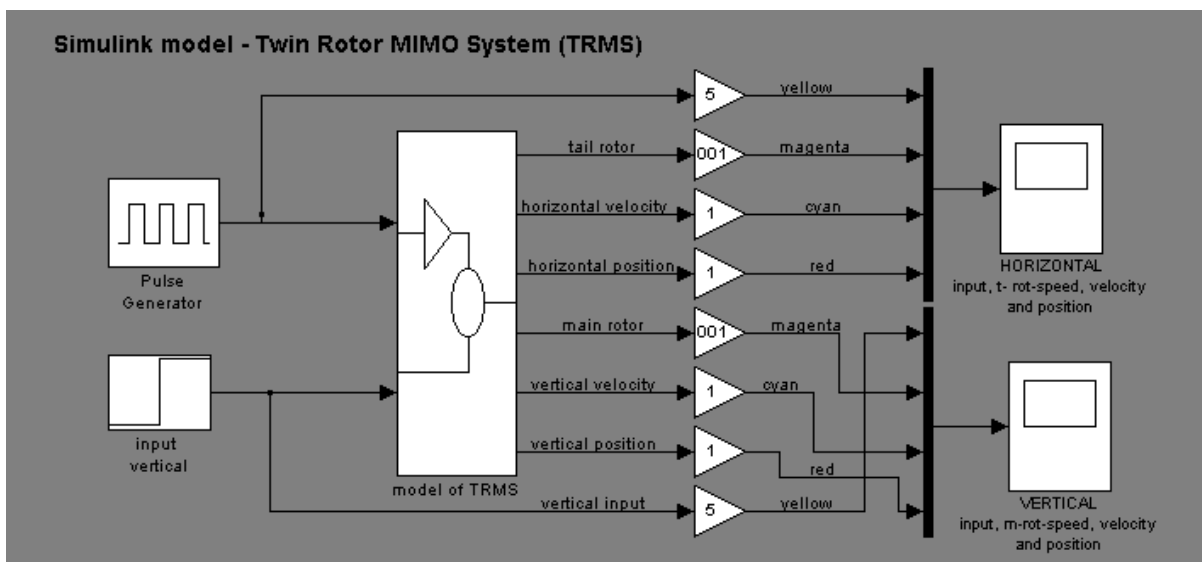


Figure 4-3: 2-DOF complex model of TRMS

Having ungrouped the block *model of TRMS* in Figure 4-3 you can see the details of it (Figure 4-4). Notice the connections between the horizontal and vertical sub-models.

The model shown was constructed and calibrated on the basis of physical laws. Therefore, it is simple to find all physical coefficients (described in section 2) in the masked or grouped blocks. On the basis of this model you can create your own models: either simplified or more complex. You can confirm the quality of new models by real-time experiments using RTK tools.

Note: It is strongly recommended to save this model in an archive directory before changing any coefficients. It is the only place where the default (pre-identified) values of the coefficients exist.

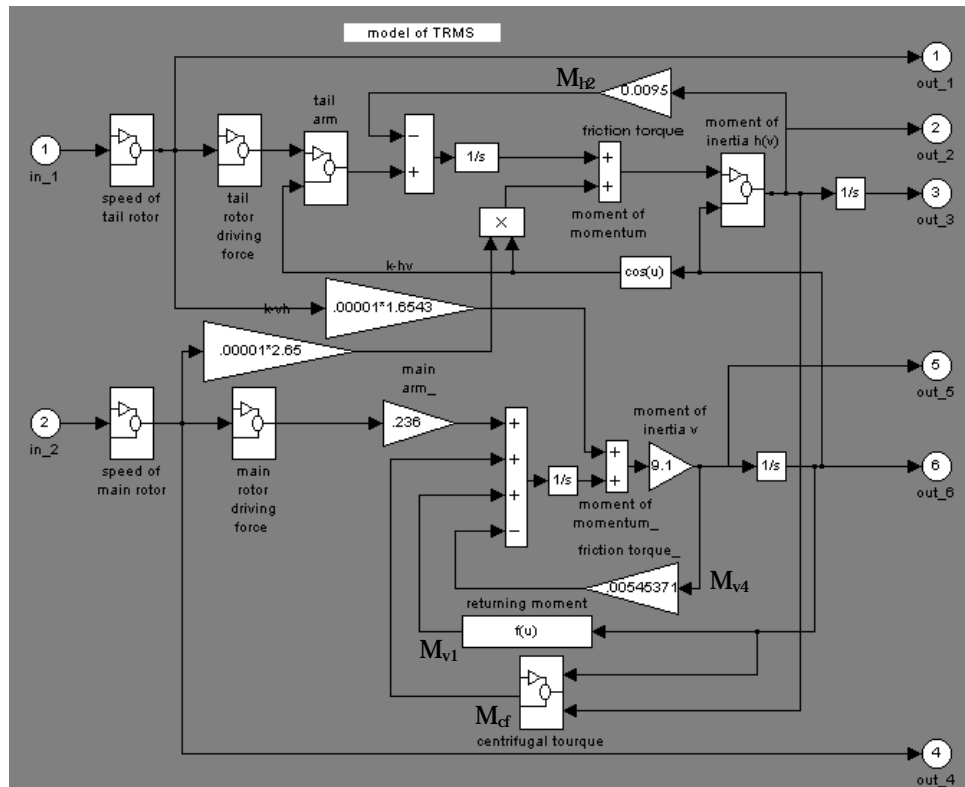


Figure 4-4: Detailed 2-DOF model of TRMS



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5. MODEL IDENTIFICATION

During identification the parameters of the mathematical model are tuned to obtain a satisfactory degree of conformity of the model with the real system. The point is to tune the parameters of the model in such a way, that the outputs of the model fit the experimental data in the sense of an assumed objective function (criterion). The most frequently used objective function is the integral square error.

$$Q = \int_0^{\infty} \varepsilon^2(t) dt = \int_0^{\infty} (y(t) - y_m(t))^2 dt$$

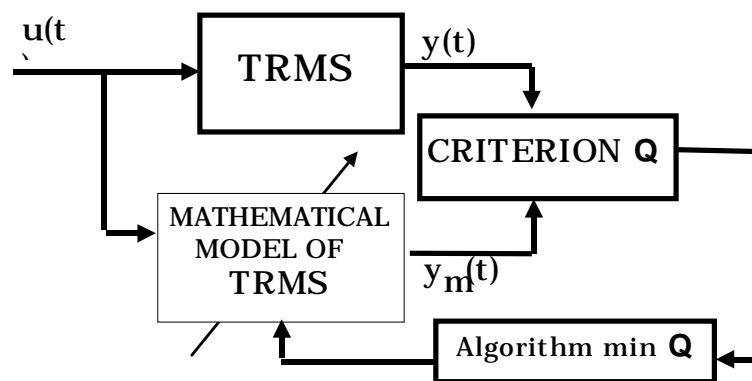


Figure 5-1: Block diagram of identification procedure.

By changing model parameters, using an appropriate minimisation method, we find the minimum value of the objective function Q and the corresponding values of model parameters.

It is a classical problem of static optimisation. The knowledge of the system structure makes possible the identification of selected parts of the model.

MATLAB offers many algorithms that can be used for this purpose; among them are two functions **fmin.m** and **fmins.m** which are very simple to use. In the case when a limited number of parameters is tuned, elementary optimisation methods can be applied.

Figure 5-2 illustrates the idea of tuning two coefficients of the model using the Gauss-Seidel algorithm, using trial and error.

Note, if available, the MATLAB Optimisation Toolbox provides a fast and accurate Gauss-Seidel optimisation method.



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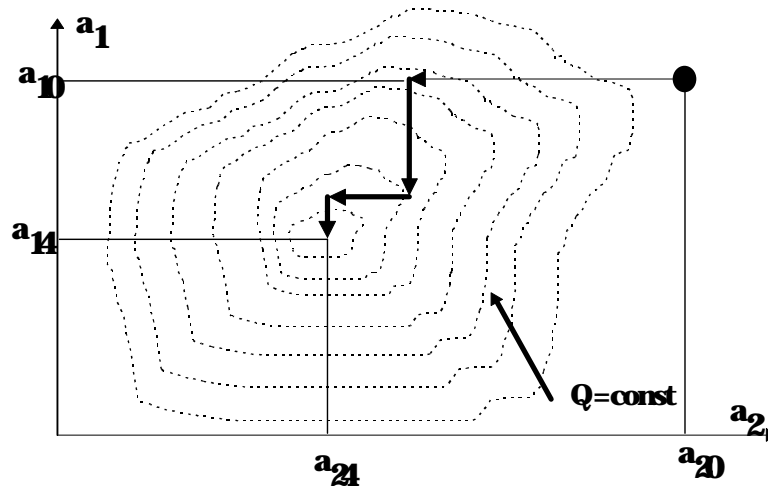


Figure 5-2: Gauss-Seidel optimisation method

It was demonstrated in Section 3.1 that most of the model coefficients can be determined using simple physical measurements. However, there are coefficients connected with friction which must be determined by dynamic experiments. They are: the *friction torque* blocks in Figure 4-4 and the time constants T_{mr} and T_{tr} in Figure 4-1c and Figure 4-2c. Other coefficients of the model can be verified using a similar method.



5.1. IDENTIFICATION OF TRMS PARAMETERS

5.1.1. Example I. Tuning parameters of the friction torque in the 1-DOF vertical model

The following experiment calculates the friction torque of the vertical motor, based on an open loop plant response and simulation.

In this experiment the beam is free to move in the vertical plane (This part of the TRMS can be treated as the identification of the parameters of a pendulum).

Before any other actions, set the TRMS system to the 1-DOF vertical plane by mechanically blocking its freedom to move in the horizontal plane (by tightening the vertical axis locking screw as shown in Figure 5-3).

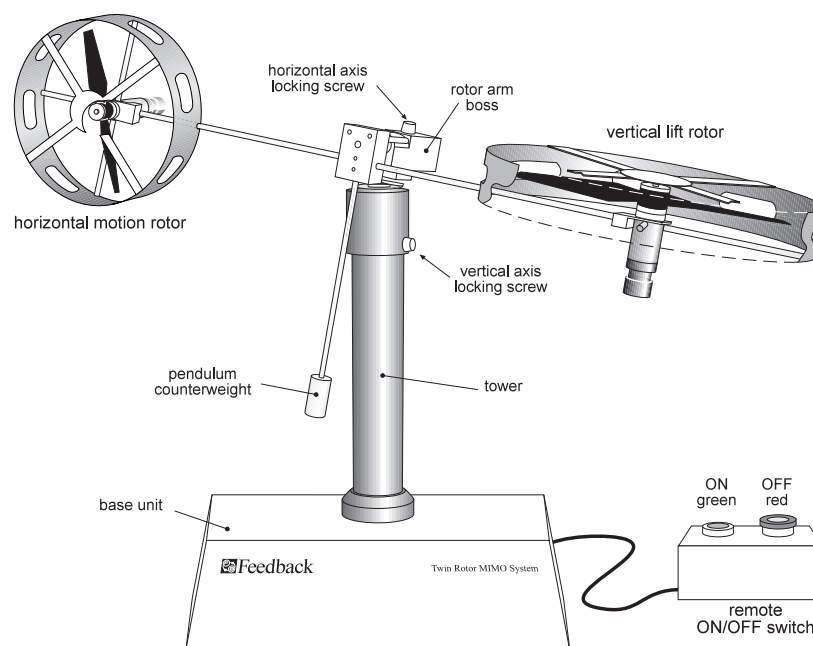


Figure 5-3. TRMS vertical and horizontal axis locking screws

Invoke **hl_1doff** from the MATLAB *Command Window*.

Set the sampling time and the initial conditions of the model as they were in the real-time experiment.

Put the beam in the horizontal position and then let it move freely. Measure and save the response of the sub-system.



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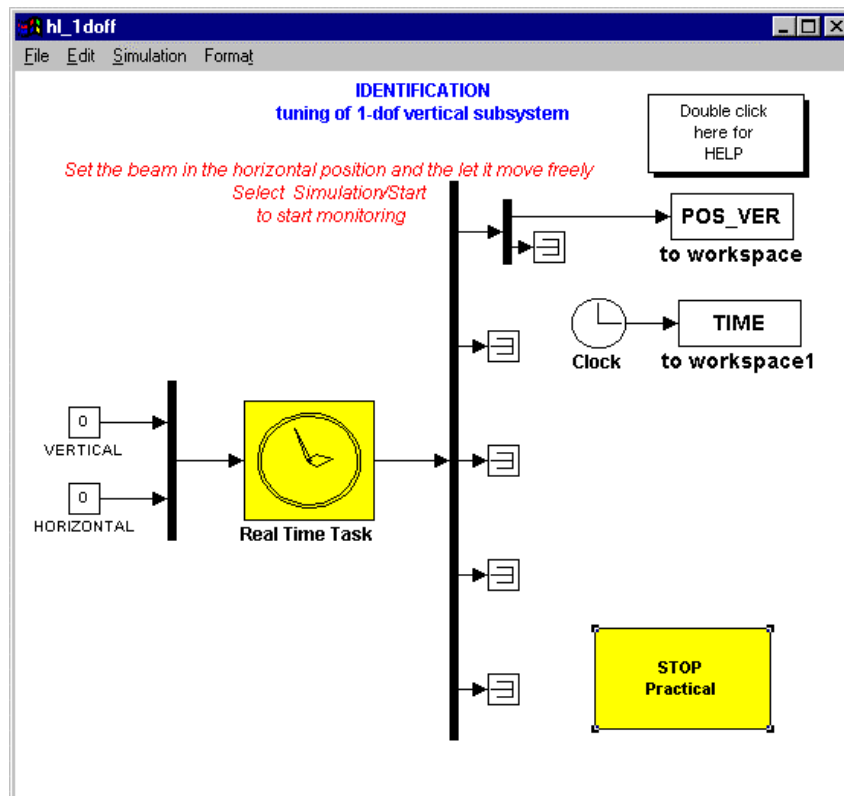


Figure 5-4: Block diagram of **hl_1doff** experiment.

Next, start the simulation part of this experiment using the Simulink model of the 1-DOF vertical system. In order to carry it open the **m_1doff** model from MATLAB *Command Window* and start simulation. Set the Simulation parameters – *Stop Time* to 100 for both experiments.

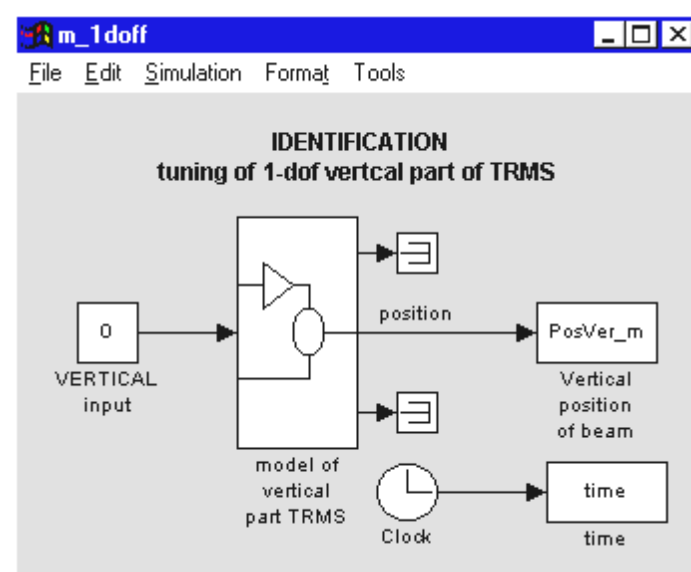


Figure 5-5: Block diagram of **m_1doff** model.



After the experiments are finished, type the following command at the MATLAB prompt

```
Trms1
```

This file collects the outputs from the last two experiments and compares them by producing plots.

Figure 5-6a shows free responses of the real system and those of the model. Notice, that the oscillation periods, damping and steady states are different.

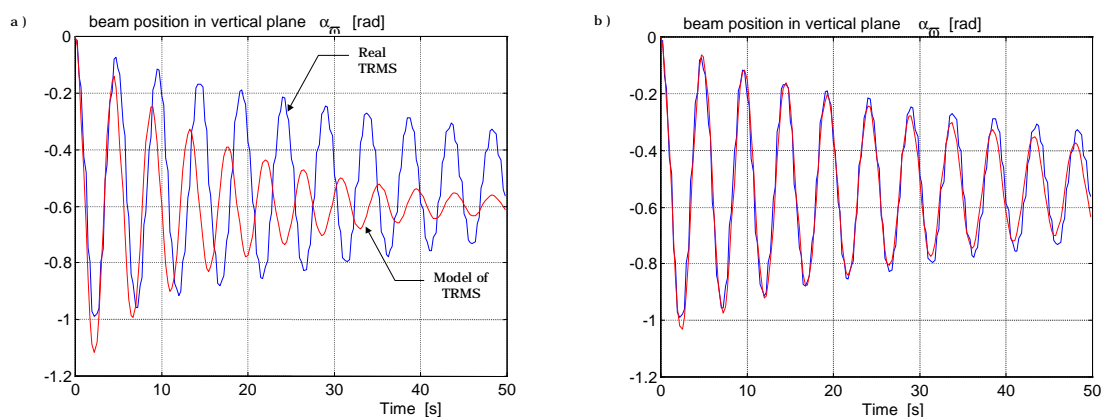


Figure 5-6: Comparison of free oscillations of the beam (model and real system) a) before and b) after identification of the model coefficients..

Now carry the Gauss-Seidel identification procedure changing coefficients connected with *friction*, *return torque* and *moment of inertia* (Figure 4-4). Figure 5-6b shows the comparison of the free responses of the system and the model, with new values of coefficients calculated after the tuning procedure.

The Gauss-Seidel identification routine could be executed much faster and with greater accuracy by using the Optimisation Toolbox from MATLAB. If this is not available, use the trial and error method.



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5.1.2. Example II. Tuning of time constants of the 1-DOF vertical motor-propeller model

Another example of identification is the determination of main-motor propeller time constant T_{mr} . First, invoke **hl_omv** from MATLAB *Command Window* and then carry experiment. In this experiment a step response of $\omega_m(t)$ is measured (see Fig.5.8.a TRMS curve).

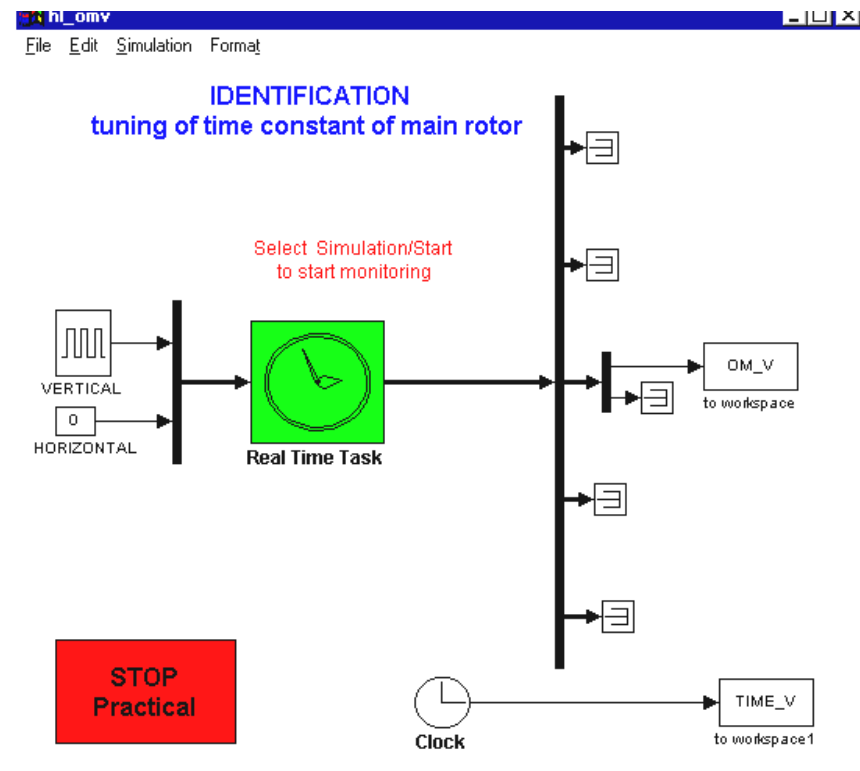


Figure 5-7: Block diagram of **hl_omv** experiment

Next, invoke **m_omv** from the MATLAB *Command Window* and run the simulation of the model.

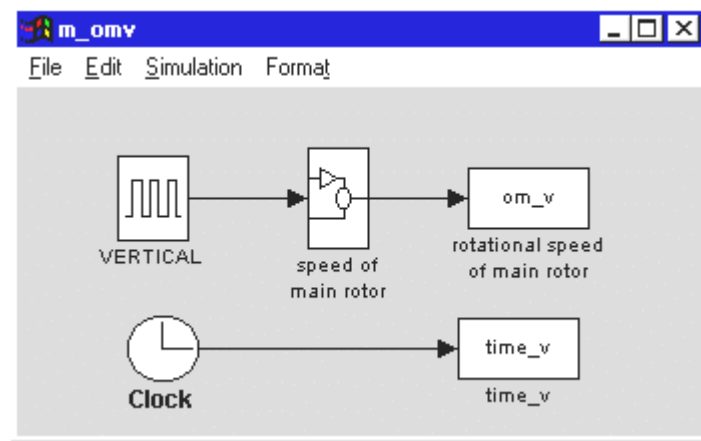


Figure 5-8: Block diagram of *m_omv* model.

If the step input response does not match the simulated one, then it is necessary to adjust the time constant in the *m_omv.m* file until the two responses give similar results. The identified value for T_{mr} in this case is 1.234 seconds. The responses of both real and simulated systems are shown in Figure 5-9, and they can be obtained by typing the following command at the MATLAB prompt.

Trms2

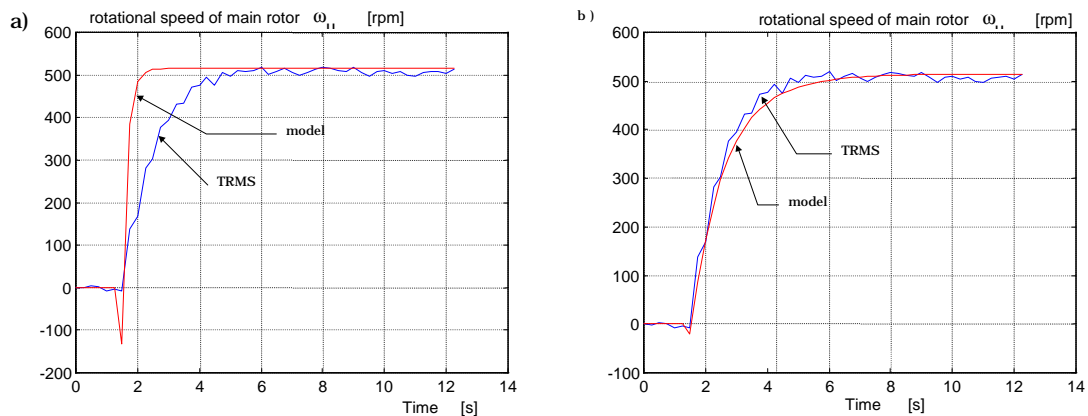


Figure 5-9: Results of time constant T_{mr} identification. Model and system step responses –
a) before b) after identification

5.1.3. Example III Tuning of Time constant for 1-DOF Horizontal propeller

This example of identification is the determination of the tail-motor propeller time constant T_{tr} . Before any other actions, set the TRMS system to the 1-DOF in the horizontal plane by mechanically blocking its freedom to move in the vertical plane (by tightening the horizontal axis locking screw as shown in Figure 5-3). First, invoke the file *hl_omh* and carry out the experiment which is similar to the previous one.



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In this experiment a step response of $\omega_t(t)$ is measured (see Figure 5-12 -a TRMS curve). Then, run the simulation of model *m_omh*.

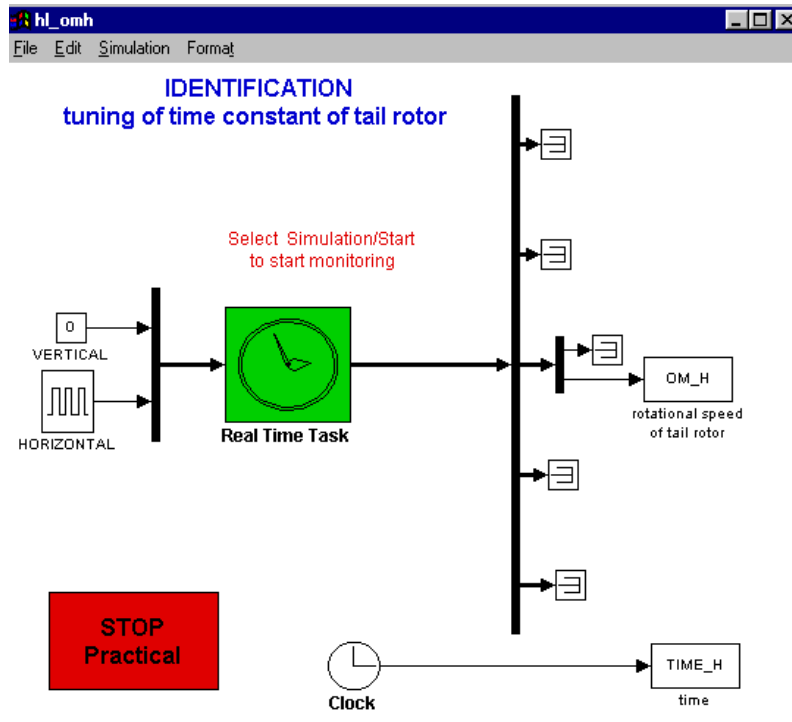


Figure 5-10: Block diagram of *hl_omh* experiment

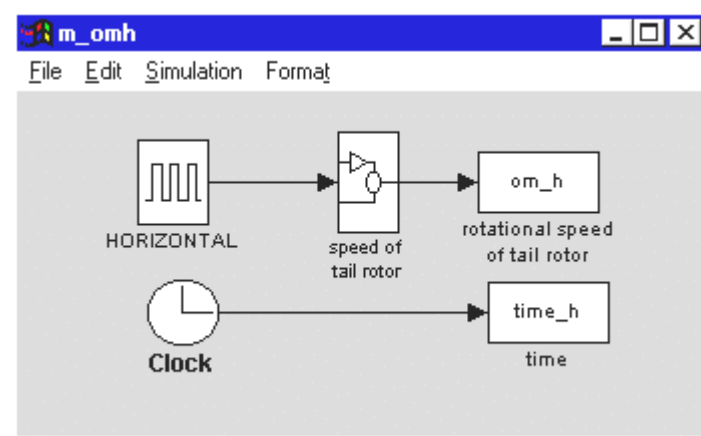


Figure 5-11: Block diagram of *m_omh* model

Figure 5-12a and Figure 5-12b shows the results of the time constant T_r identification for the tail rotor, and they can be obtained by typing the following command at the MATLAB prompt.

Trms4

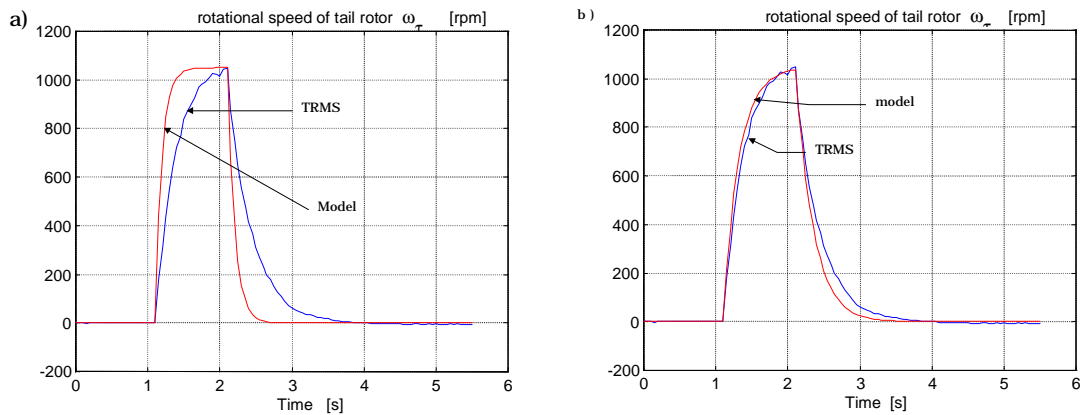


Figure 5-12: Results of the time constant T_{tr} identification. Model and system step responses
a) before b) after identification

Identification procedures are very time consuming but it is necessary to carry them out precisely. Design of the controllers is more effective when a reliable identified mathematical model is used.

If you obtain results that do not match the simulated model, this could be caused by a change of the rotation of the propeller in an opposite direction, leading to negative values which need subtracting.

The best approximation of the real time constant of the tail rotor is given by: $\frac{0.25}{0.3842s + 1}$



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6. TUNING OF PID CONTROLLER

In general, tasks of a TRMS control system can be listed as follows:

- **stabilisation.** Design a controller so that the state vector of the closed-loop system is stabilised around a desired point of the state space.
- **tracking.** Design a controller so that the closed-loop system output follows a given trajectory.

The control problems considered in this section are the following. Design a PID controller that will stabilise the position of the TRMS, or make it to follow a desired trajectory in one plane (1-DOF) while motion in the other plane is blocked. If the TRMS is free to move in both axes we refer to the control as two degrees of freedom (2-DOF problem). The description of the PID controllers applied for TRMS is given in the TRMS Getting Started Manual - 33-007-1M5.

Four control systems are considered and the appropriate files can be used to model the behaviour of closed-loop systems:

- 1-DOF PID control system - horizontal plane (file: **ss_1dofh.mdl**)
- 1-DOF PID control system - vertical plane (file: **ss_1dofv.mdl**)
- 2-DOF Simple PID control system (file: **ss_2dofs.mdl**)
- 2-DOF Cross-coupled PID control system (file: **ss_2dofc.mdl**)

You can invoke any of these control systems by typing the appropriate name of the system in the MATLAB *Command Window* or by using the *Models & Control Systems* button in *Main Control Window* (see Figure 6-1).

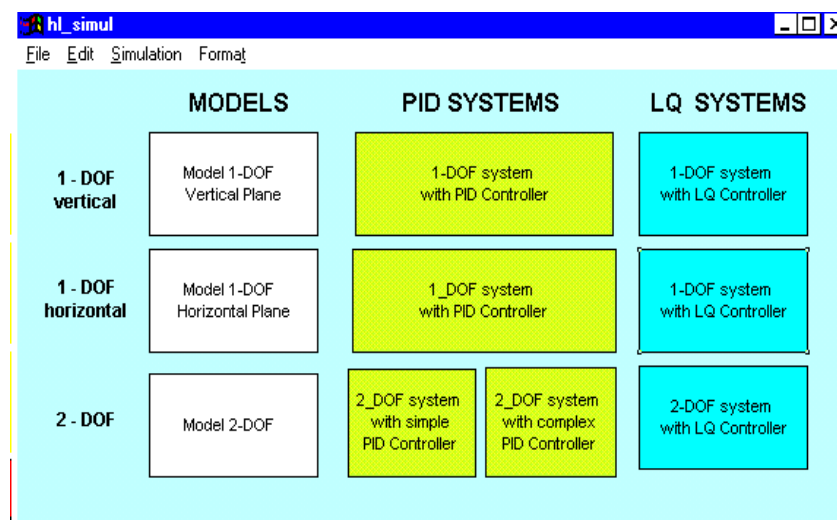


Figure 6-1: *Models & Control Systems* window



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The control system models can be used for design of PID controllers (by any method). After calculating the PID coefficients you can study the behaviour of the modelled closed-loop systems and then confirm the quality of control by real-time experiments.

There are several methods of designing of closed-loop control systems. In order to obtain optimal (or sub-optimal) settings of parameters for PID controllers the so-called tuning methods can be used. The following tuning methods can be distinguished:

- tuning (time or frequency based). An experiment is performed with the process and with the model of the process. Tuning rules are based on time or frequency responses of the system.
- more general is the method of an objective function minimisation. The idea of this method for TRMS with a PID controller is shown in Figure 6-2.

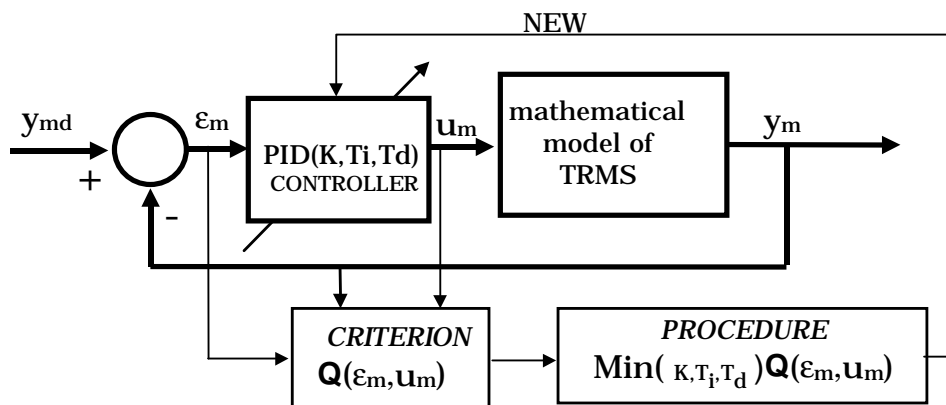


Figure 6-2: Schematic diagram of tuning of PID parameters (for TRMS)

The most commonly used criterion's are functions of square error ε^2 . The following criterion was used for the experiments described below:

$$Q = \int_0^{\infty} \varepsilon_m^2(t) dt$$

Notice, that selection of the criterion is a rather complicated and difficult task and is closely related to the project assumptions.

Optimal values of the coefficients of PID controllers must be passed to RTK in order to start the experiments (see section 6.5).



6.1. PARAMETER TUNING OF PID CONTROLLER FOR A 1-DOF HORIZONTAL SYSTEM

The tuning of PID controller parameters is performed using the Simulink model **ss_1dofh** (1-DOF system with PID controller button in 1-DOF horizontal row in Figure 6-1) and a Gauss-Seidel minimisation procedure. The beam is allowed to move only in the horizontal plane. Figure 6-3 shows the Simulink model used in this case, and Figure 6-4 shows the structure of the PID controller. Note that the PID controller uses only the position signal from the TRMS model.

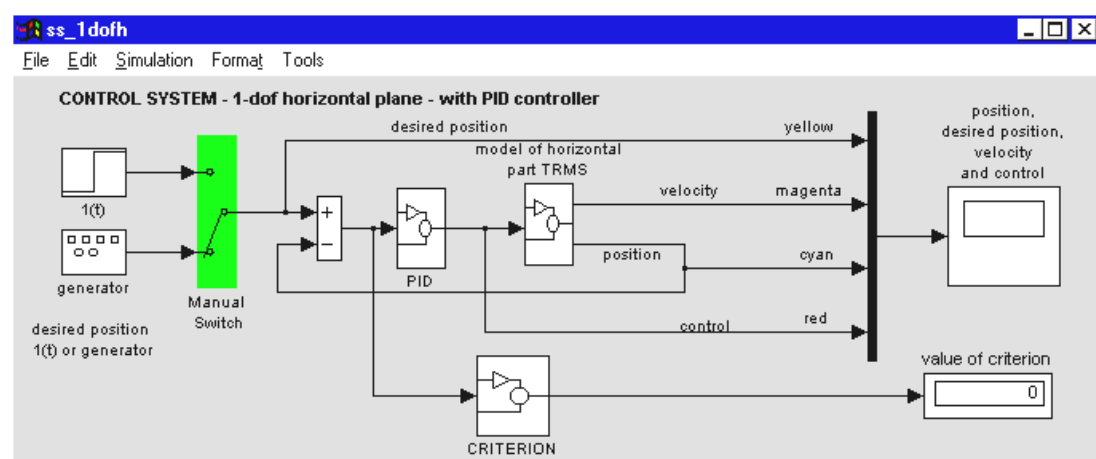


Figure 6-3: Simulink model of 1-DOF horizontal PID control system

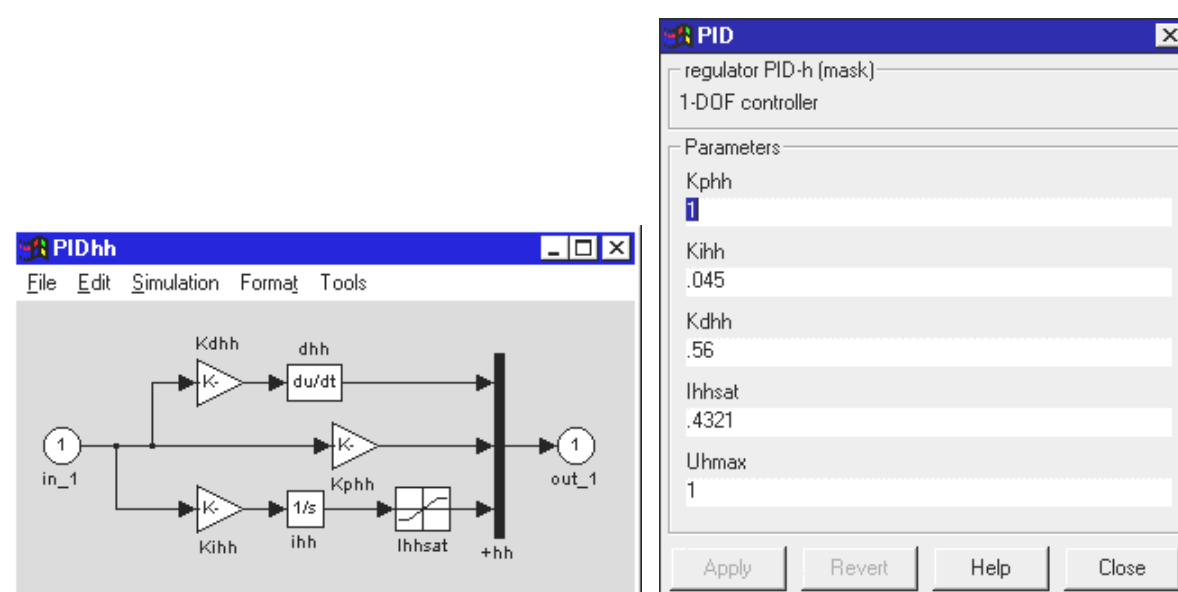


Figure 6-4: Structure of PID controller and its mask



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To obtain optimal values of the PID controller parameters the following steps should be performed:

1. - invoke **ss_1dofh** model,
2. - open *PID* block,
3. - click *Simulation*,
4. - change parameters of PID controller in PID block according to Gauss-Seidel algorithm,
5. - observe value of criterion,
6. - if this value is minimal finish tuning,
7. - otherwise go back to 3.

Figure 6-5 shows the response of the model to a square wave reference input having previously completed the parameters tuning procedure.

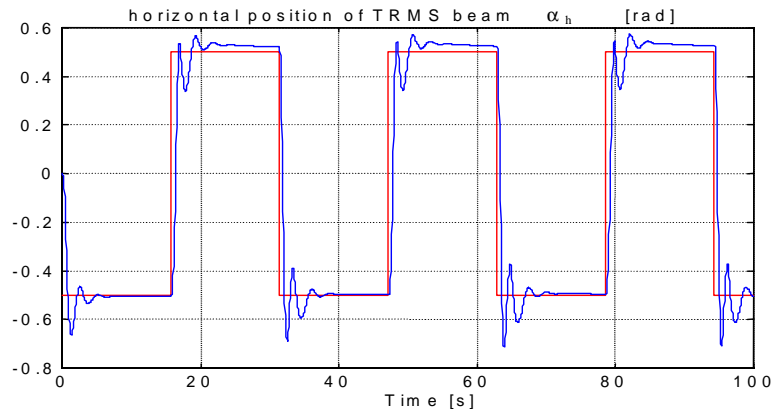


Figure 6-5: 1-DOF horizontal simulation for $Q = \int_0^{\infty} \epsilon_m^2(t) dt$ tuning criterion

The selected values of parameters of the PID controller can be transferred to RTK (see section 6.5). Figure 6-6 shows the results of a real-time experiment.

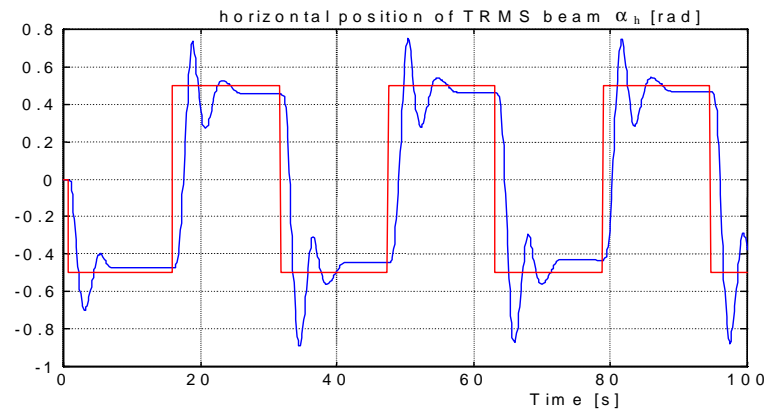


Figure 6-6: 1-DOF horizontal experiment for $Q = \int_0^{\infty} \varepsilon_m^2(t) dt$ tuning criterion

6.2. PARAMETER TUNING OF PID CONTROLLER FOR 1-DOF VERTICAL SYSTEM

The tuning of PID controller parameters is performed using the Simulink model **ss_1dofv** (1-DOF system with PID controller button in 1-DOF vertical row in Fig.6.1) and Gauss-Seidel minimisation procedure. The beam is allowed to move only in the vertical plane. As before, the PID controller uses only the position signal from the TRMS model.

The tuning procedure includes the following steps:

1. - invoke **ss_1dofv** model,
2. - open *PID* block,
- ➔ 3. - click *Simulation*,
4. - change parameters of PID controller in PID block according to Gauss-Seidel algorithm,
5. - observe value of criterion,
6. - if this value is minimal finish tuning,
- 7. - otherwise go back to 3.

Figure 6-7 shows the response of the model to a square wave reference input after the minimisation procedure was completed.



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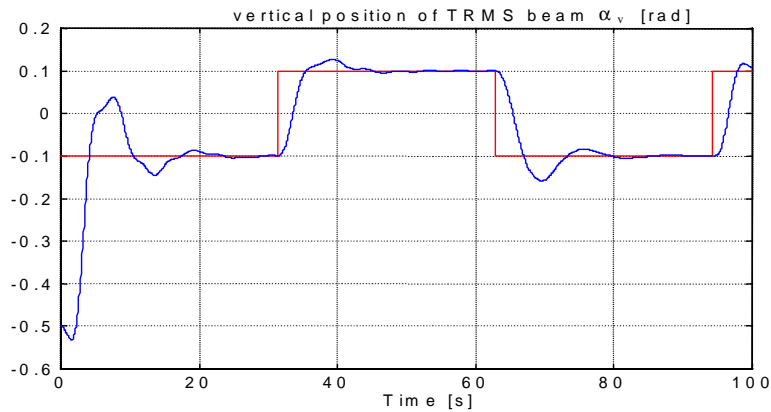


Figure 6-7: 1-DOF vertical simulation. Criterion: $Q = \int_0^{\infty} \varepsilon_m^2(t) dt$

The optimal values of parameters of the PID controller can be passed to RTK. Figure 6-8 shows the results of a real-time experiment.

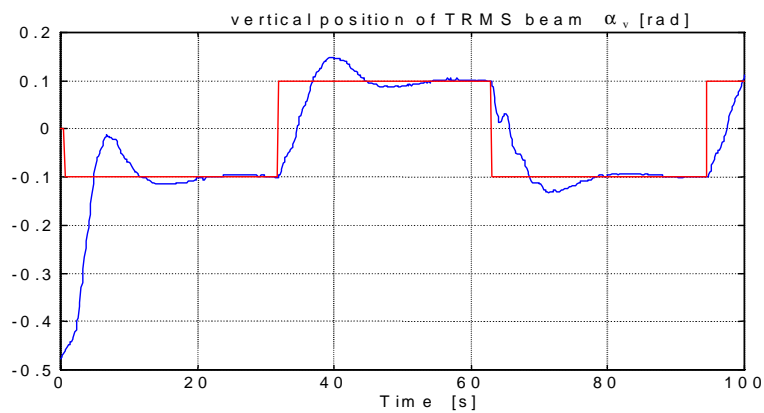


Figure 6-8: 1-DOF vertical experiment for $Q = \int_0^{\infty} \varepsilon_m^2(t) dt$ tuning criterion



6.3. PARAMETER TUNING FOR 2-DOF SYSTEM WITH SIMPLE PID CONTROLLER

In this case the beam can move in both planes. Using Simulink model **ss_2dofs** (2-DOF system with simple PID controller button in Fig.6.1) and minimisation procedure, as described in Section 6.1, parameters tuning of PID controller can be performed. To obtain optimal values of the PID controller parameters the following steps should be performed:

1. - invoke **ss_2dofs** model,
2. - open *PID-hh* and *PID-vv* block,
3. - click *Simulation*,
4. - change parameters of PID controller in *PID* blocks according to Gauss-Seidel algorithm,
5. - observe value of criterion,
6. - if this value is minimal finish tuning,
7. - otherwise go back to 3.

Figure 6-9 shows the Simulink model and Figure 6-10 shows the structure of the simple PID controller used during this experiment.

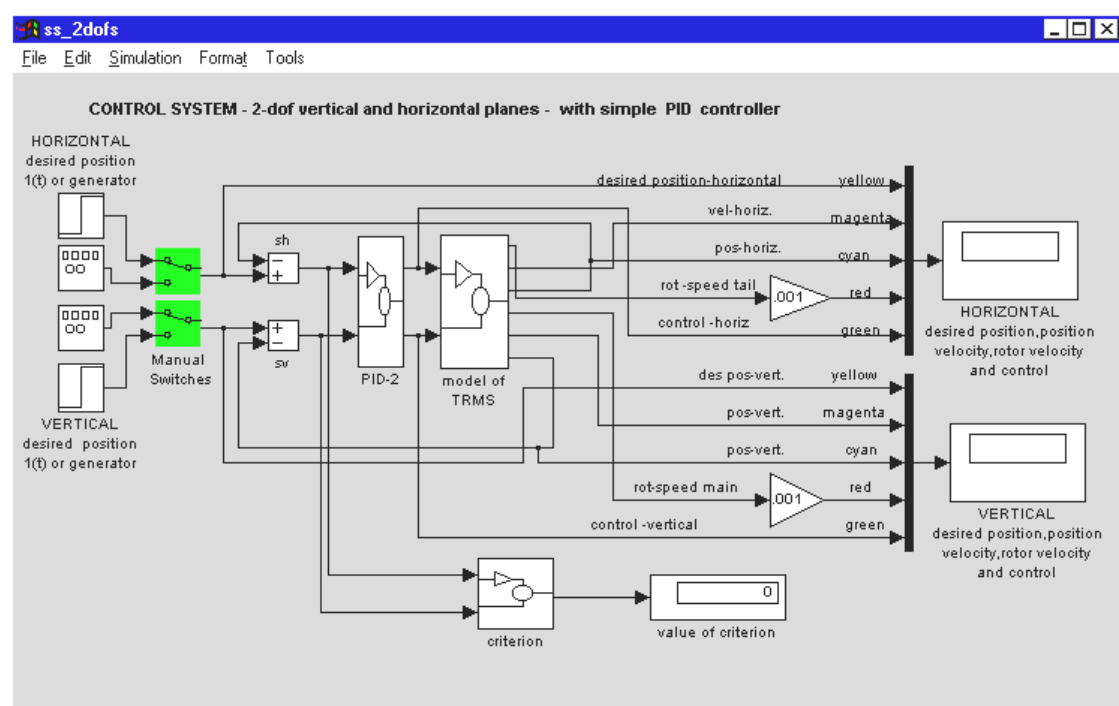


Figure 6-9: Simulink model of 2-DOF with simple PID controller



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Structures of the blocks *PID-vv* and *PID-hh* in Figure 6-10 are identical as in Figure 6-4. Figure 6-11 shows the responses of the model after the minimisation procedure. The reference inputs are: a sinusoidal wave in the vertical plane and a square wave in the horizontal plane. Notice, the strong interaction between the step change of input in the horizontal plane and response in the vertical plane.

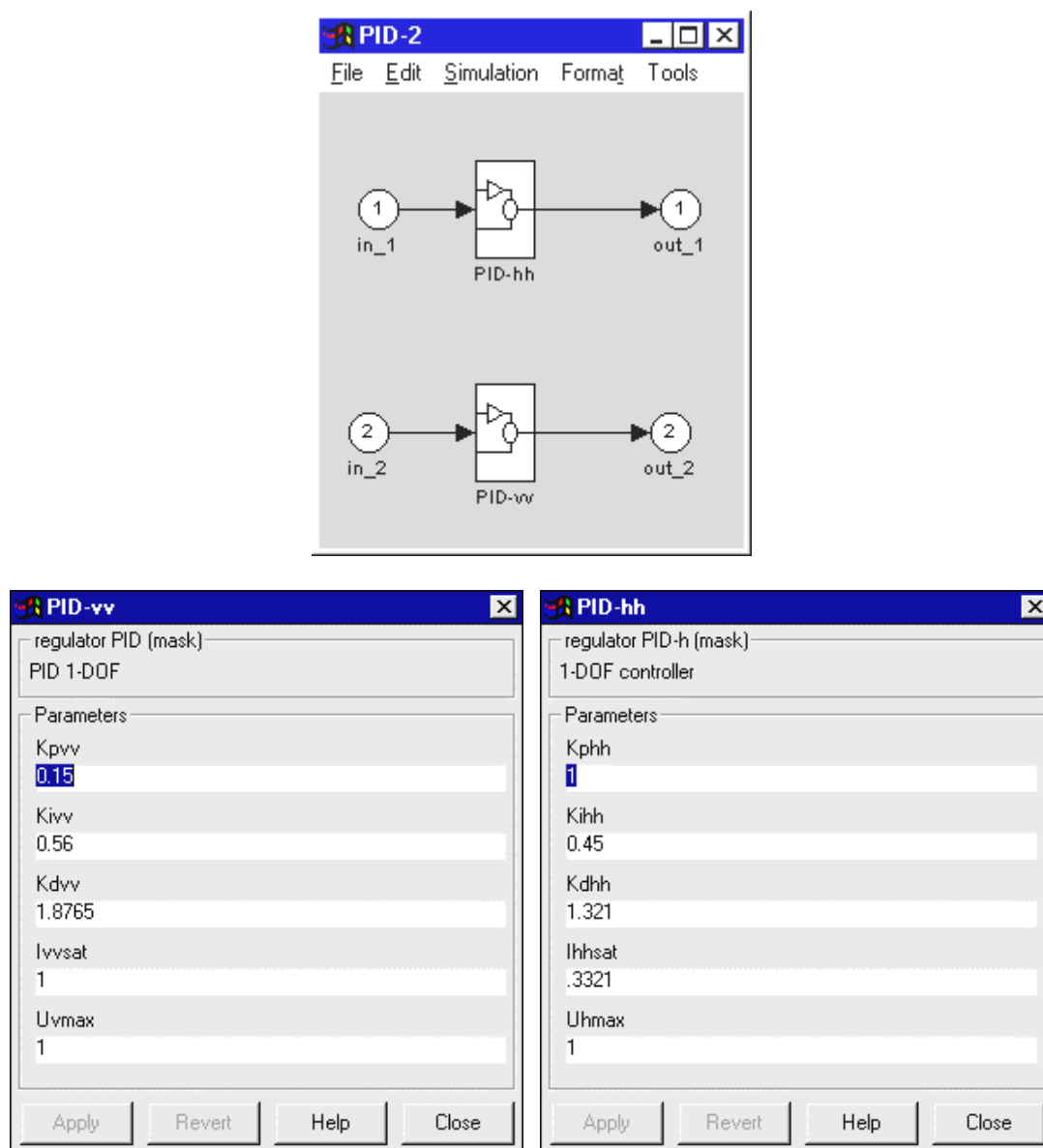


Figure 6-10: Structure and masks of the simple 2-DOF PID controller

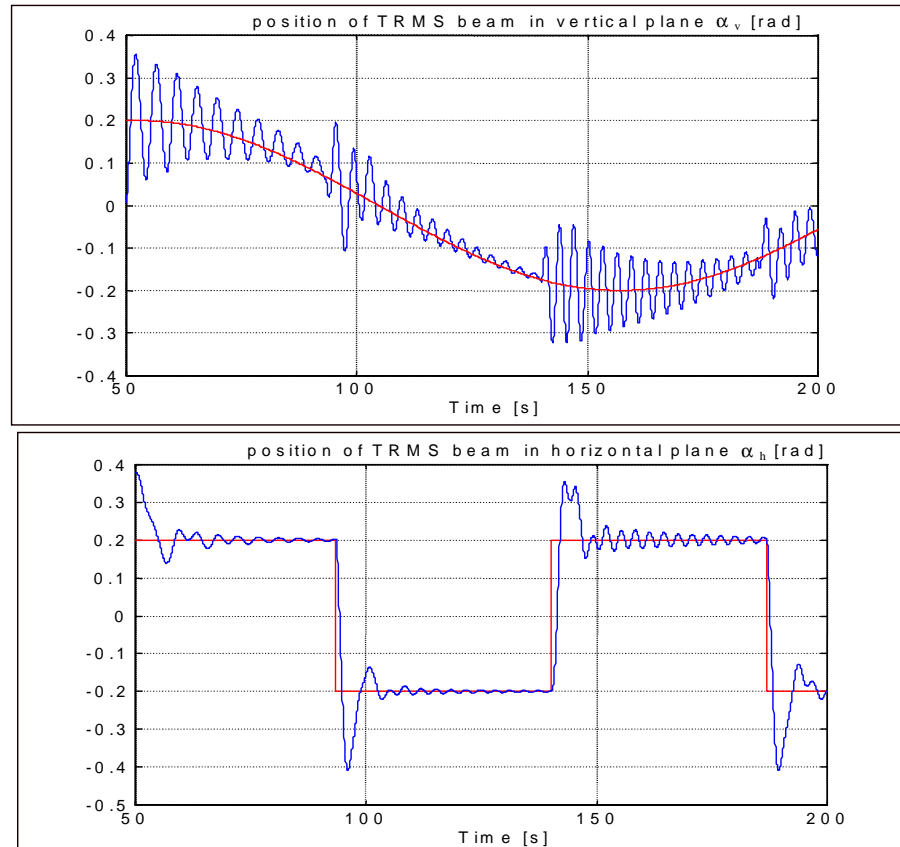


Figure 6-11: Results of simulation for 2-DOF system with simple PID controller

Fig.6.12 shows the responses of the TRMS with the PID parameters passed to RTK in a real-time experiment. The reference inputs have the same shapes as in the simulation experiment. Interactions between the vertical and horizontal responses are also seen.



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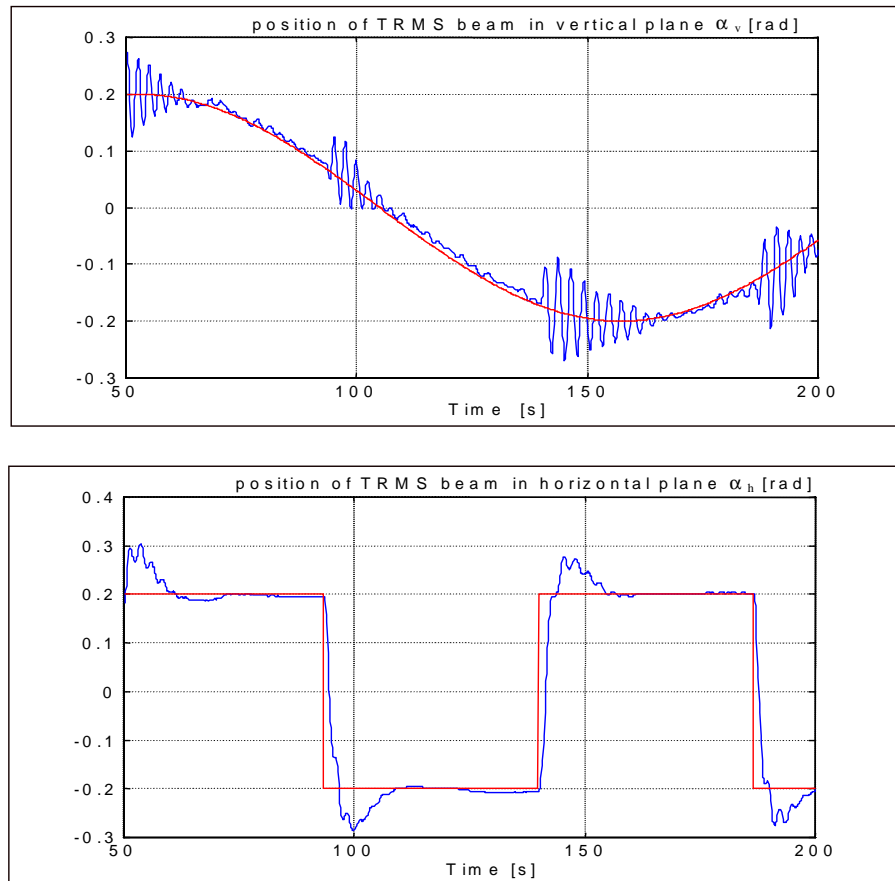


Figure 6-12: Results of experiment with 2-DOF system with simple PID controller



6.4. PARAMETER TUNING OF THE 2-DOF SYSTEM WITH A CROSS-COUPLED PID CONTROLLER

All simulations and real-time experiments described in this section are similar to those described previously. The only difference is in the PID controller structure. In the present case the Simulink model **ss_2dofc** (*2-DOF system with complex PID controller* button in Figure 6-1) is used.

Figure 6-13 shows the model of the closed-loop system. Figure 6-14 shows the block diagram of the cross-coupled PID controller. This structure of the controller was proposed for reducing interactions between motions in both planes. The equations of the TRMS system are of the sixth order but can be naturally separated into two three-order sets representing the horizontal and the vertical motion, respectively.

Unfortunately, there is an important amount of interaction between the vertical and horizontal motion. The goal of uncoupling of the system is to design a controller such that a change of reference value for pitch will cause only a vertical movement of the beam with no change of azimuth (the beam position). A similar effect in the other axis is expected when a de-coupling controller is applied.

To obtain optimal values of the PID controller parameters the following steps should be performed:

1. - invoke **ss_2dofc** model,
2. - open *PID-vv*, *PID-hh*, *PID-vh* and *PID-hv* blocks,
- 3. - click *Simulation*,
4. - change parameters of PID controller in *PID* blocks according to Gauss-Seidel algorithm,
5. - observe value of criterion,
6. - if this value is minimal finish tuning,
- 7. - otherwise go back to 3.



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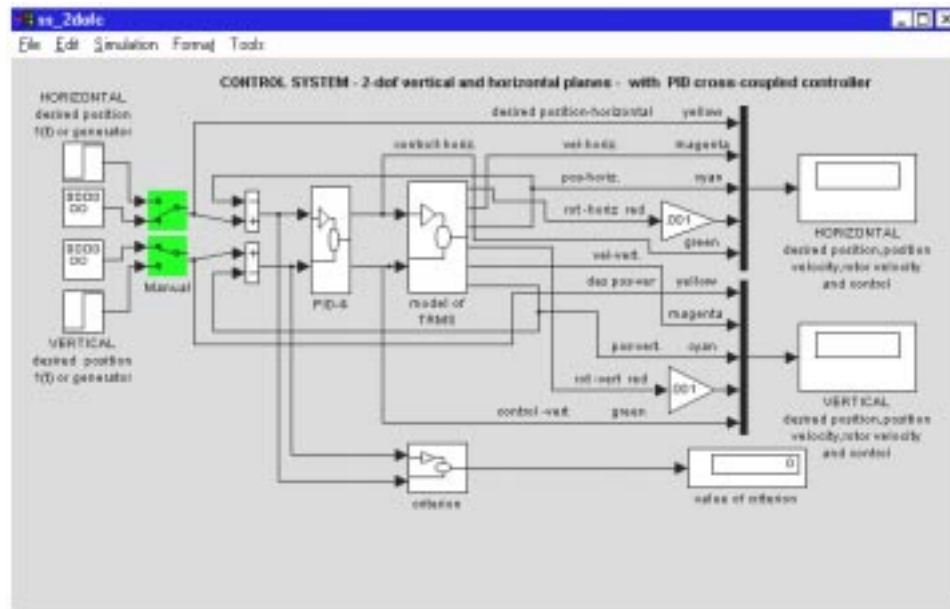


Figure 6-13: Simulink model of 2-DOF with cross-coupled PID controller

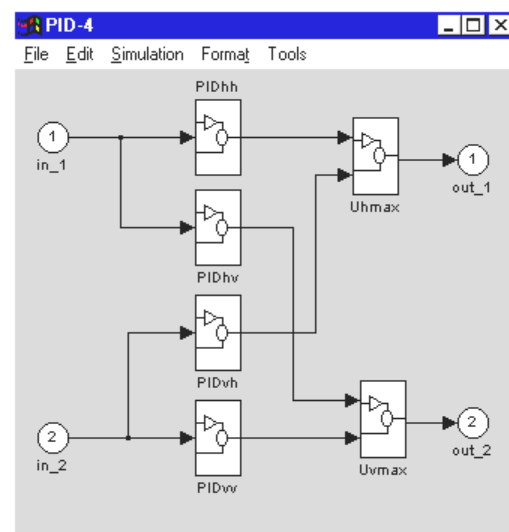


Figure 6-14: Block diagram of the cross-coupled PID controller

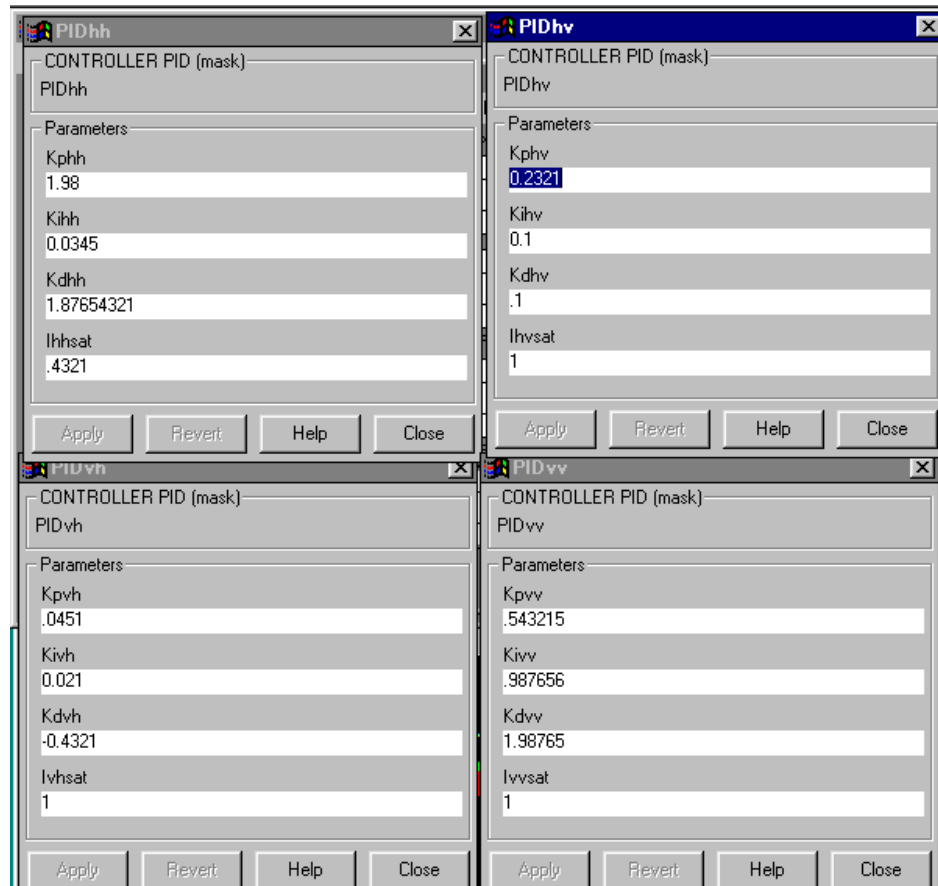


Figure 6-15: Structure of cross-coupled PID 2-DOF controller and masks of this controller



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Figure 6-16 shows responses of the model after the minimisation procedure has been completed. The reference inputs are a sinusoidal in the vertical plane and a square wave in the horizontal plane. Notice that the influence of the step change in the horizontal plane on the response in the vertical plane is weaker than with the simple PID controller.

This effect was achieved by experimental tuning of 18 parameters of the 2-DOF cross-coupled PID controller.. Figure 6-16 shows the responses of the TRMS in the real-time experiment with the parameters of PID previously passed to RTK. The Reference inputs have the same shape as in the simulation experiment. Weak interactions between the horizontal and vertical responses can be found in Figure 6-16 and Figure 6-17.

Note: Theoretical aspects of non-linear MIMO system de-coupling will be considered in the TRMS Advanced Teaching Manual 2 – 33-007-5M5.

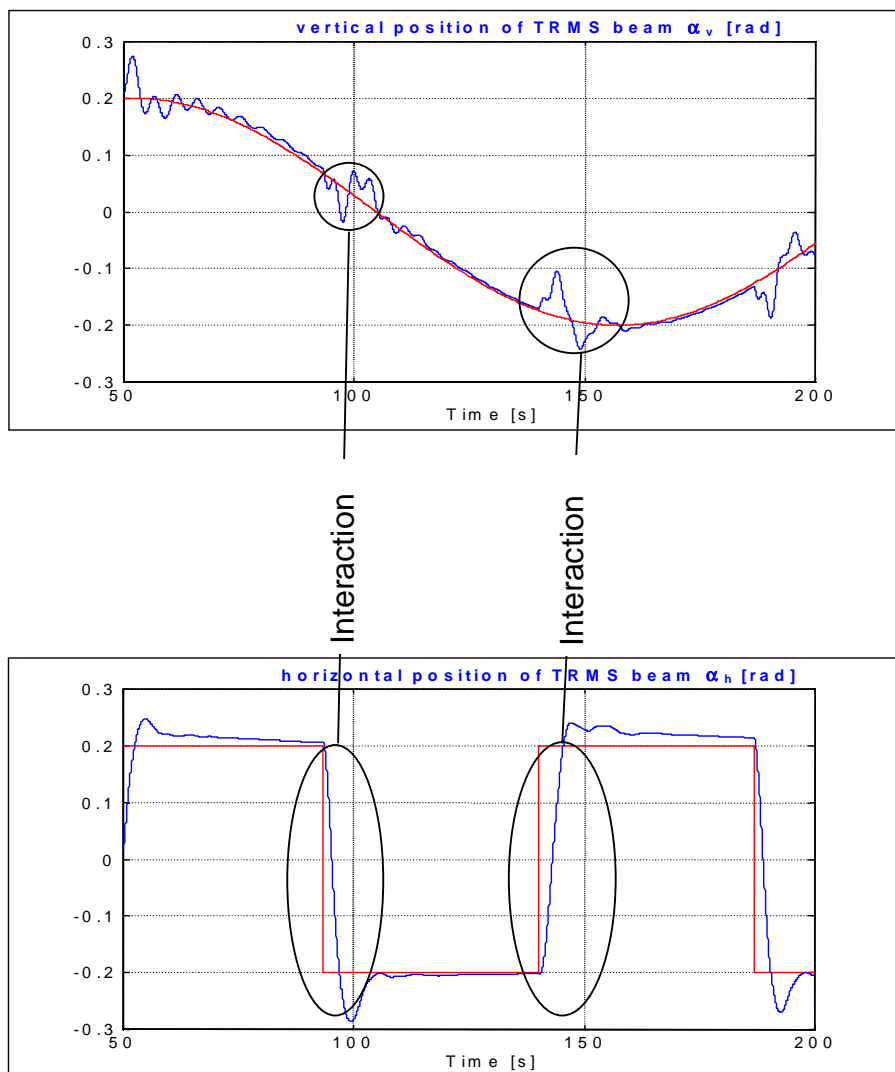


Figure 6-16: Results of simulation (2-DOF system with cross-coupled PID controller)

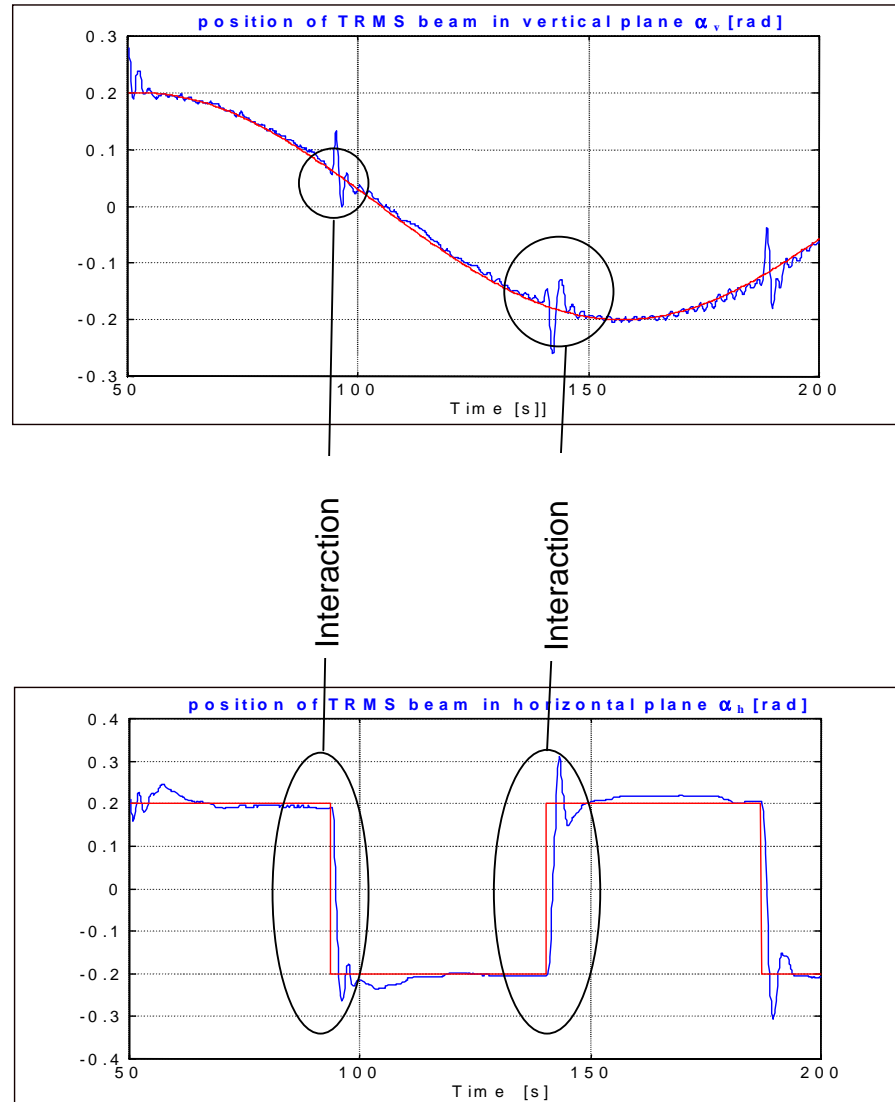


Figure 6-17: Results of experiment (2-DOF system with cross-coupled PID controller)



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6.5. PARAMETER TRANSFER TO RTK

There are two ways of transferring the parameters to the RTK.

Method 1

The first one uses the *Set controller parameters* window. A special communication window was constructed, for transferring the values of the parameters to RTK (Figure 6-18), and this can be opened any time by clicking the block *Controller Parameters* window.

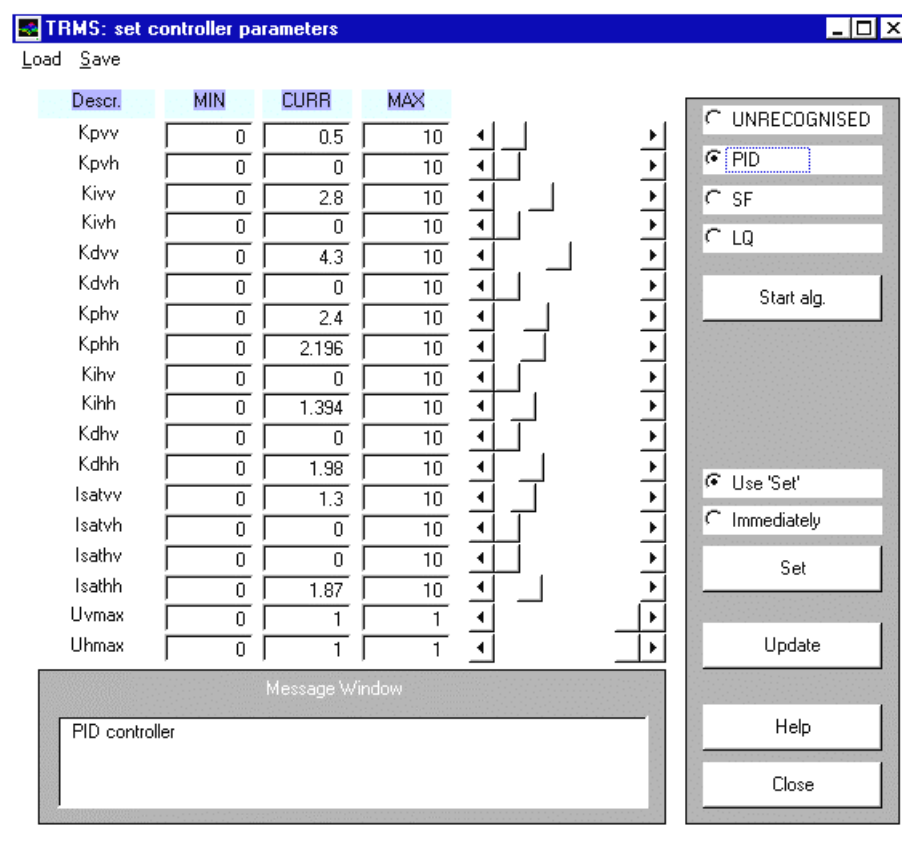


Figure 6-18: *Set controller parameters* window for PID controllers

If an arbitrary algorithm is running and next this window is opened then the algorithm is marked (the black dot) with the current active values of parameters. If none of the algorithms are running, then the UNRECOGNISED button is marked without any visible parameters.

You can select the type of the algorithm by clicking the PID, SF or LQ radio-button. Each algorithm has its own set of default parameters, which can be changed by typing an appropriate value or by dragging the slider.



When the *Use 'Set'* button is selected the parameters are passed to the RTK after clicking *Set* button. When the *Immediately* button is selected, each change is passed to the RTK immediately.

Any set of parameters previously saved can be loaded using the *Load* menu option. The default parameters are loaded from the *hl_cpar.mat* file for inactive algorithms, whilst for the active algorithm (working in the background) the parameters are loaded from the RTK.

The *Update* button is used to show the current parameters of the RTK. If you click the *Update* button then the parameters window is updated. It means that current values of parameters from RTK are transmitted to the parameters window. Therefore all changes introduced lately outside the visible window, for example: in the MATLAB Command Window, etc are transmitted to that window.

If you mark a control algorithm and click the *Start alg.* button then the new control algorithm will be chosen and it will start.

Method 2

The second way to setting the controller parameters is to type appropriate values directly to the mask of the *Real Time Task* block in the Simulink model. (Figure 6-19).

In this mask you can set all 18 parameters of the PID controller. After the start of the experiment, these values of the parameters will transfer to the RTK, if the check box is marked, otherwise the parameters set in this block will not be updated to the "Controller Parameters" window.



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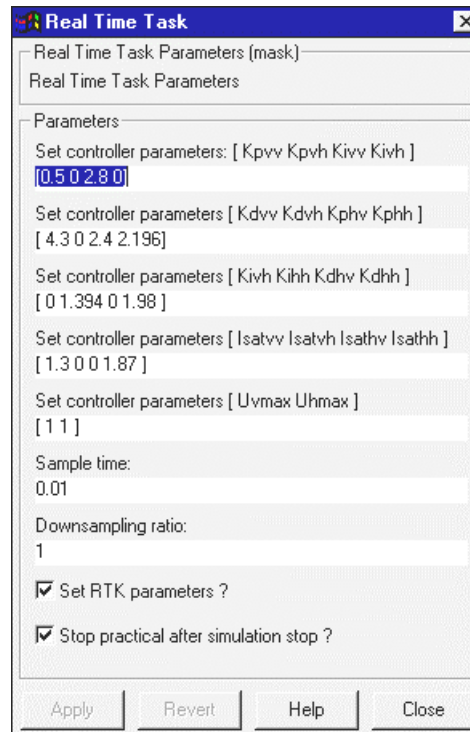


Figure 6-19: Mask of *Real Time Task* in the PID stabilisation case.

The procedure described above, is relevant for all real-time experiments.

It is also possible to insert the values calculated for the PID controller parameters into either of the tables shown in Figure 6-18 and Figure 6-19, and compare the results with the theoretical (simulated) ones.



7. LQ-SYSTEMS FOR TRMS

The synthesis of the continuous LQ controller for TRMS is described below. For a very small value of the sampling time the response of the discrete system converges to the response of the corresponding continuous system, which is the case we require.

The most important question for the designer of a control system, as far as LQ regulator problem is concerned, is how to select the weighting factors of the cost function.

The linearised dynamic model of TRMS is described by the linear differential equations in the general form:

$$\begin{aligned}\frac{dx}{dt} &= Ax + Bu \\ y &= Cx\end{aligned}$$

where the matrices A,B,C are obtained using appropriate MATLAB function (see next section).

A typical quadratic cost function has the form:

$$J(u) = \int_0^{\infty} [x^T(t)Qx(t) + u^T(t)Ru(t)]dt$$

if the following conditions are fulfilled:

- the matrix $Q \geq 0$, Q is nonnegative definite,
- the matrix $R > 0$, R is positive definite,
- the (A,B) pair is controllable.

The weighting matrices Q and R are selected by a designer but they must satisfy the above conditions. It is most easily accomplished by picking the Q 's to be diagonal with all diagonal elements positive or zero. Some positive weight ($|R| \neq 0$) must be selected for the control, otherwise the solution will include infinite control gains.

The values of elements of Q and R matrices are weakly connected to the performance specification. A certain amount of trial and error is required with a simulation program to achieve satisfactory result. A few guidelines can be recommended. For example, if all states are to be kept under close regulation and Q are diagonal with entries so selected that a fixed percentage change of each variable makes an equal contribution to the cost. The matrix R is also diagonal.



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Then the optimal scalar control u^* and the optimal trajectory vector x^* are given by:

$$u^* = -K x^*$$

where K is the feedback matrix.

The optimal control problem is now defined as follows: find the gain K such that the feedback law minimises the cost function subject to the state equation. The calculation of the control variable which minimises the criterion is a dynamic optimisation problem. This problem can be solved by variational calculus applying the maximum principle due to the Bellman optimisation principle. The procedure returns the optimal feedback matrix K , the matrix S , the unique positive definite solution to the associated matrix Riccati equation:

$$SA + A^T S - SBR^{-1}B^T S + Q = 0$$

Because of the quadratic appearance of S , there is more than one solution, and S must be positive definite to select the correct one. The procedure returns also the matrix E , the closed-loop roots:

$$E = \text{eig}(A - BK)$$

The vector K can be calculated by a numeric iterative formula on the basis of the Riccati equation. The associated closed-loop system:

$$dx/dt = (A - BK) x$$

is asymptotically stable.

To solve the LQ controller problem the **lqry** function can be used from the *Control System Toolbox*. The synopsis of **lqry** is: $[K, S, E] = \text{lqry}(A, B, C, D, Q, R)$.

The control u is not constrained. Such assumption can not be satisfied for a real physical system. One must remember that if the control u is during practical saturated then the obtained control is no more of the LQ type. To return to the LQ problem the amplitude of the u signal should be diminished. In such a case the designer tunes the relative weights between state and control variables. As far as it happened the simulation tools are recommended to be applied.



7.1. LQ CONTROL OF 1-DOF HORIZONTAL SYSTEM

7.1.1. Simulation model for LQ design

The simulation model shown in Figure 7-1 is used as the basis of the design of the LQ controller for a 1-DOF horizontal system

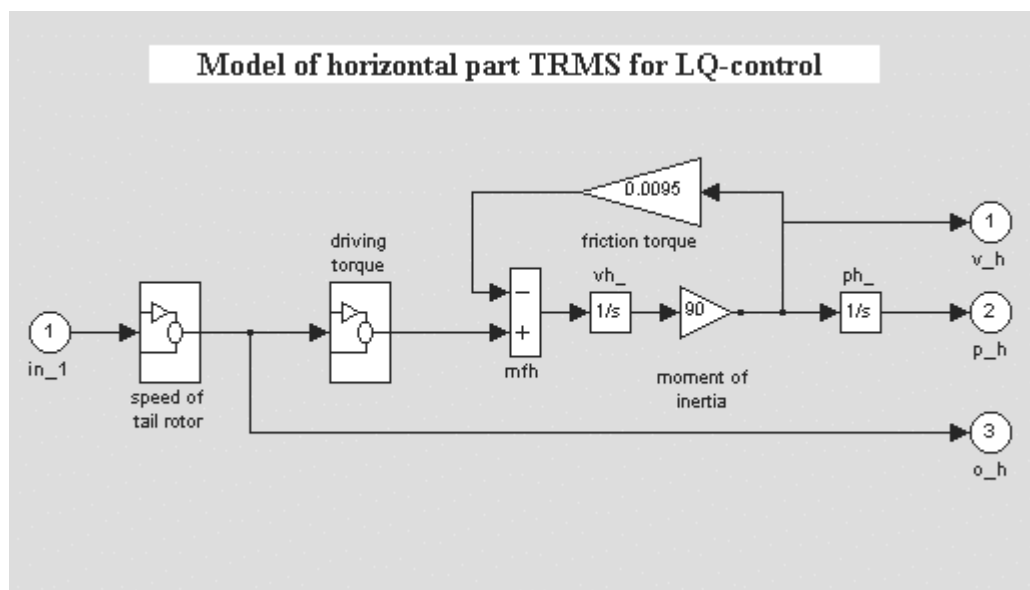


Figure 7-1: Model of the horizontal part TRMS applied for LQ-control

The design process comprises the next steps:

- linearise , in fixed point, the non-linear model presented above using *linmod* MATLAB function,
- select Q and R matrices,
- calculate feedback gains using *lqry* function,
- simulate the closed-loop LQ system with linear model and check the behaviour of the system,
- if it is necessary, change Q or R and repeat the simulation,
- simulate the closed-loop non-linear system,
- perform a real-time experiment with the feedback gains obtained.



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7.1.2. Calculating LQ feedback gains for 1-DOF horizontal

First double click the *Models & Control Systems* block, followed by double clicking the *1-DOF System with SF Controller* for 1-DOF horizontal. The appropriate window for LQ-controller will appear (see Figure 7-2).

| Khh-vel | Khh-pos : | Khh-rot : |
|----------|-----------|-----------|
| | | |
| Q(1,1) : | Q(1,2) : | Q(1,3) : |
| 0.01 | 0 | 0 |
| Q(2,1) : | Q(2,2) : | Q(2,3) : |
| 0 | 1 | 0 |
| Q(3,1) : | Q(3,2) : | Q(3,3) : |
| 0 | 0 | 300 |
| | R : | |
| | 50 | |

Solve LQ problem

LQ linear model

LQ non linear model

HELP

CLOSE

Figure 7-2: LQ-design and simulation window for 1-DOF horizontal system.

Now, write in the edit fields Q and R the elements of the matrix Q (i,j) and the scalar R.. Click *Solve LQ Problem*. In the edit fields Khh-vel, Khh-pos and Khh-rot the optimal gains of LQ-controller will appear after a short time (see Figure 7-3).

| Khh-vel | Khh-pos : | Khh-rot : |
|----------|-----------|-----------|
| 0.14306 | 0.14142 | 0.00046 |
| Q(1,1) : | Q(1,2) : | Q(1,3) : |
| 0.01 | 0 | 0 |
| Q(2,1) : | Q(2,2) : | Q(2,3) : |
| 0 | 1 | 0 |
| Q(3,1) : | Q(3,2) : | Q(3,3) : |
| 0 | 0 | 300 |
| | R : | |
| | 50 | |

Solve LQ problem

LQ linear model

LQ non linear model

HELP

CLOSE

Figure 7-3: LQ-design and simulation window for 1-DOF horizontal system after clicking *Solve LQ problem*



7.1.3. Linear LQ system simulation

To carry out the simulation experiment for the linearised mathematical model with LQ controller click *LQ-linear* button in *LQ-design and simulation* window. Fig.7.4 shows the simulation diagram applied for this purpose.

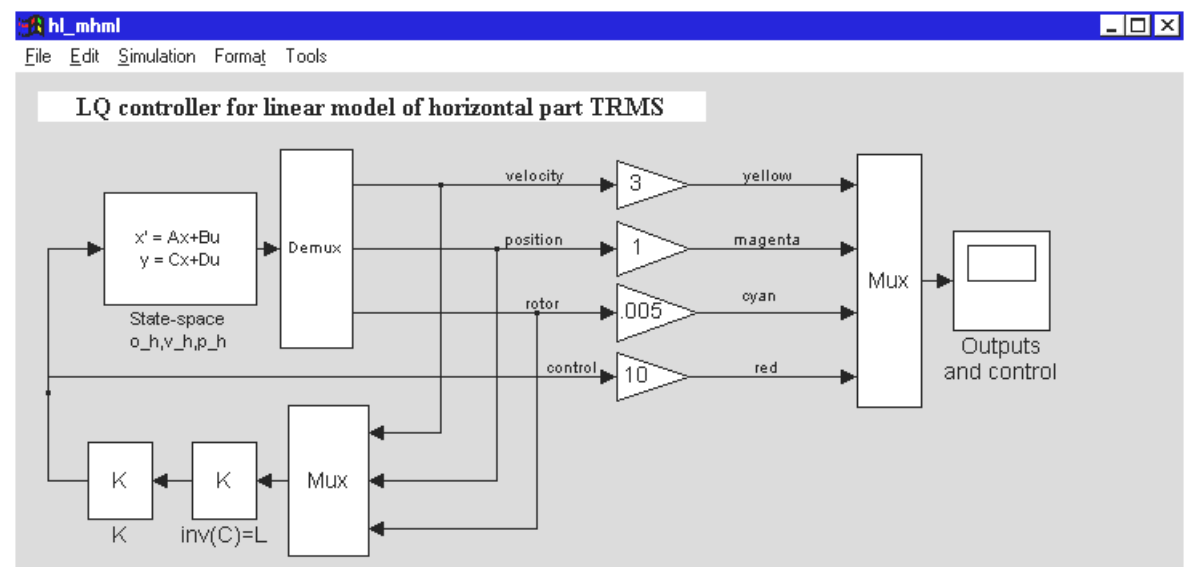


Figure 7-4: Simulation diagram for LQ-system for 1-DOF horizontal part of TRMS (linearised)

Matrices A,B,C,D of linear model were calculated before and they are available in the *WorkSpace*. In the *State-Space* block (see Figure 7-4.) you must enter the initial conditions for the state variables (the succession of states is defined in the variable **xstr** in *WorkSpace*) and the mask of the *State-Space* block is shown in Figure 7-5).

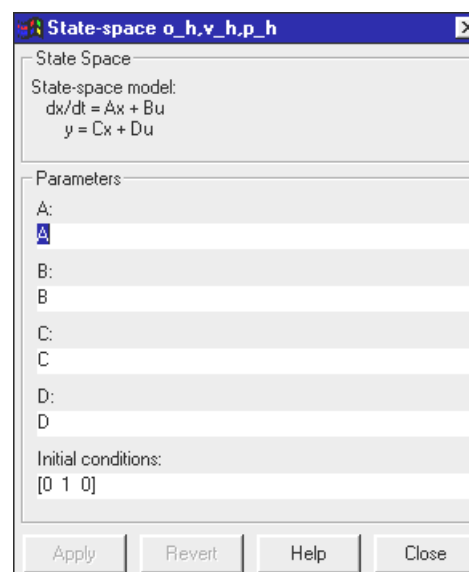


Figure 7-5: Mask of the *State-Space* block for initial condition definition of linear model of TRMS



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Values of the state variables are reconstructed from output variables in the block $INV(C)$ $=L$. The LQ closed-loop system responses are the results of a simulation experiment from initial conditions to zero point.

Remember that the system is linear and that the state variables are deviations between non-linear system states and the point of linearisation. See Figure 7-6.

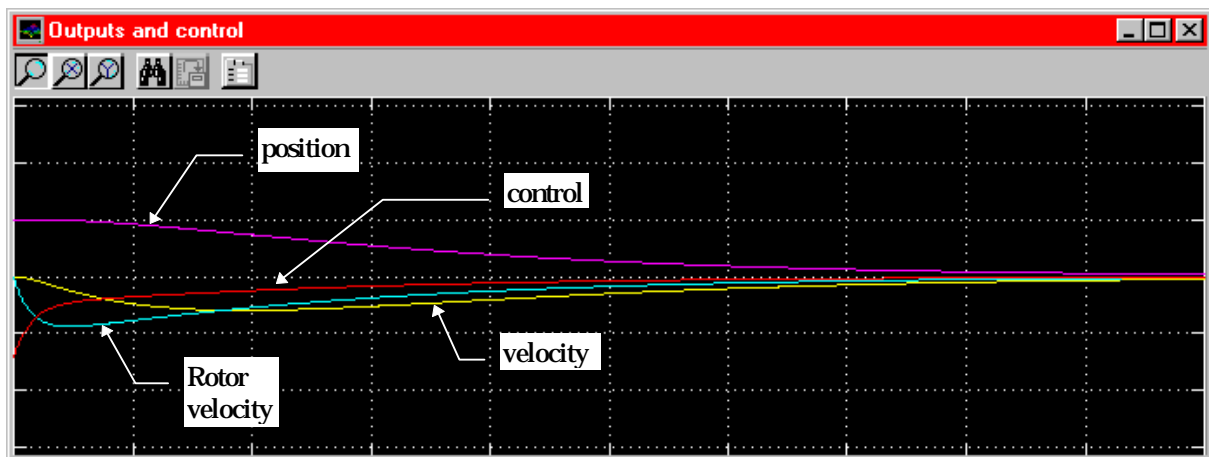


Figure 7-6: Results of the simulation of the linear TRMS model with LQ controller

You can calculate eigenvalues of the open linear system by typing in the MATLAB *Command Window*:

```
> eig(A)
```

MATLAB responds with:

```
ans =          0 ,   -0.8550 ,   -2.6042
```

and to see the eigenvalues of the closed-loop system, located in the matrix E:

```
> E
E =  -3.5679,   -1.7087 + 2.3511i,   -1.7087 - 2.3511i
```

Note that the open system is unstable, but that the LQ closed-loop system is asymptotically stable. Note also that the open system has just one eigenvalue equal to zero, connected with the position state variable. This means that all the positions are identical from the point of view of linearisation. In other words, the matrix A obtained from the minimisation procedure, is independent of the choice of linearisation point.

If your results are as expected, go to the next step and click *LQ nonlinear model* in *LQ-design and simulation* window .

If not, repeat *Solve LQ problem* (button in *LQ-design and simulation* window) using **different** Q or R matrices.



7.1.4. Non-linear LQ system simulation and results of real-time experiment

Double click *LQ non-linear model* to show the Simulink model of the LQ non-linear control system (Figure 7-7).

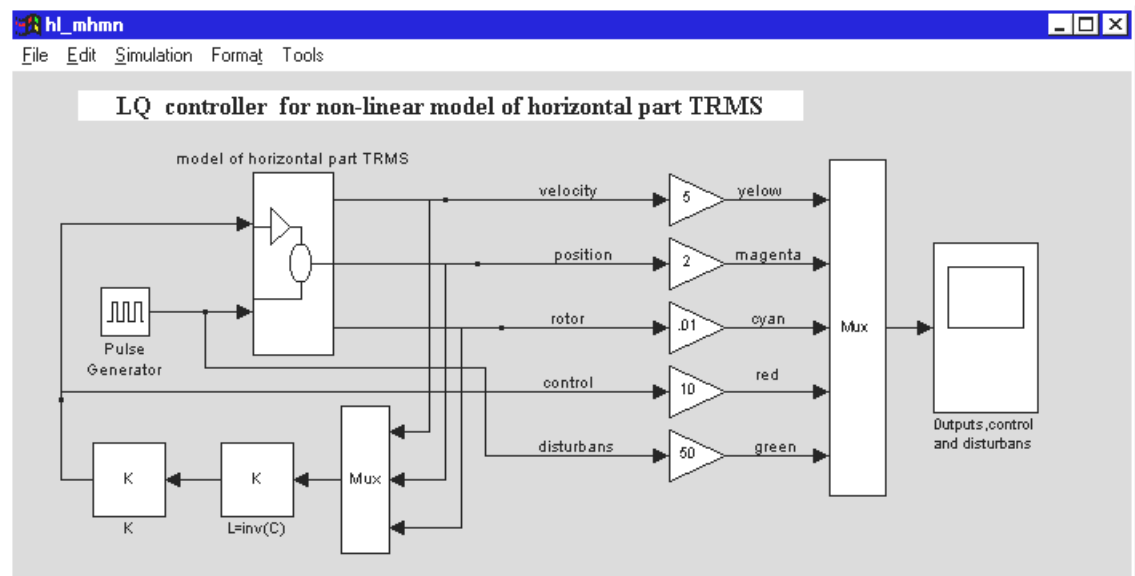


Figure 7-7: LQ controller with the non-linear model of TRMS (1-DOF horizontal part of TRMS)

Using the Simulink model of the non-linear LQ system, you can study the system responses to an external disturbance (for instance a strong blast of wind). Observe how the system comes back to the starting point (see Figure 7-8).

Note that appropriate initial condition must be typed in the integrators in *model horizontal part of TRMS*, after ungrouping this block.

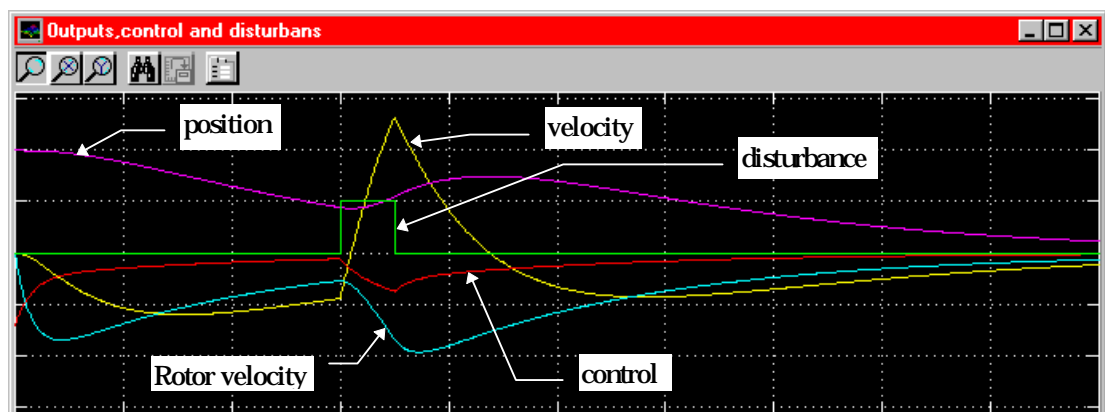
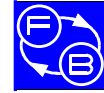


Figure 7-8: Results of simulation of the LQ controller with non-linear TRMS model (with impulse disturbances)



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After the simulation you can start a real-time experiment. - remember that this experiment will apply a stabilising LQ controller in the horizontal plane only.

At the beginning block the system in one axis so that it cannot move in the vertical plane (by tightening the appropriate screw).

Next prepare the calculated parameters to be transferred to RTK. according to the procedure described in section 6.5.

Click *Controller parameters* block and choose the LQ radio-button (see Fig.7.9).

Then type in values of K_{hh-vel}, K_{hh-pos}, K_{hh-rot} and U_{h-max} parameters. Set all others to zero.

To change the parameter set, click the *Set* button in the *Controller parameters* window. The controller now uses the default values, which can be changed by the sliders.

| Descr. | MIN | CURR | MAX |
|----------|-----|---------|-----|
| Kvv-pos | 0 | 0 | 10 |
| Kvv-vel | 0 | 0 | 10 |
| Kvv-rot | 0 | 0 | 10 |
| Kvh-pos | 0 | 0 | 10 |
| Kvh-vel | 0 | 0 | 10 |
| Kvh-rot | 0 | 0 | 10 |
| V-offset | 0 | 0 | 10 |
| Khv-pos | 0 | 0 | 10 |
| Khv-vel | 0 | 0 | 10 |
| Khv-rot | 0 | 0 | 10 |
| Khh-pos | 0 | 0.14142 | 10 |
| Khh-vel | 0 | 0.143 | 10 |
| Khh-rot | 0 | 0.00046 | 10 |
| H-offset | 0 | 0 | 10 |
| Uvmax | 0 | 0 | 1 |
| Uhmax | 0 | 1 | 1 |

Figure 7-9: *Controller Parameters* window for 1-DOF horizontal real-time experiment with LQ controller

When the system is stabilised, push the TRMS beam by hand in the horizontal plane and observe the behaviour of the system.



Next double click *LQ Controller* block in the *Main Control Window*.

The window shown in Figure 7-10a opens.

Next open the mask of *Real Time Task* (see Figure 7-10b) and set the appropriate LQ controller parameters values (these values now taken as the default).

Click *Simulation/Start* button. - the experiment starts with the values of the parameters taken from the mask of the *Real Time Task*.

The results of this experiment are shown in Figure 7-11.

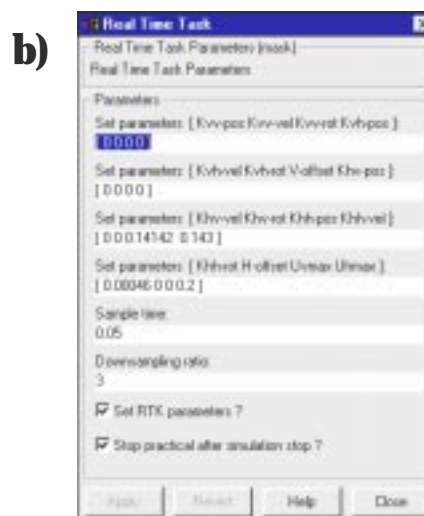
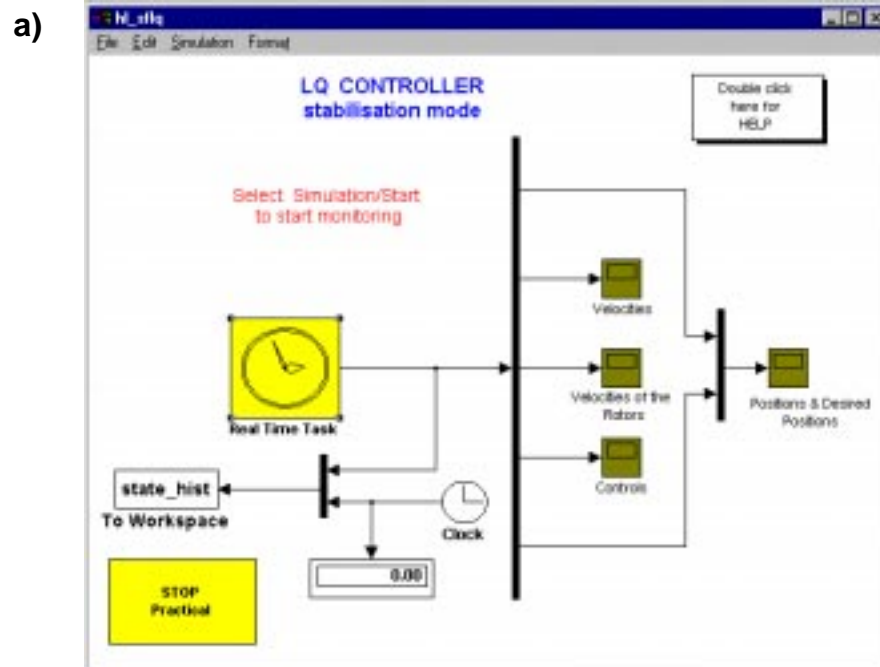


Figure 7-10: a).Real-time experiment window for LQ controller b). *Real Time Task* mask for the LQ controller



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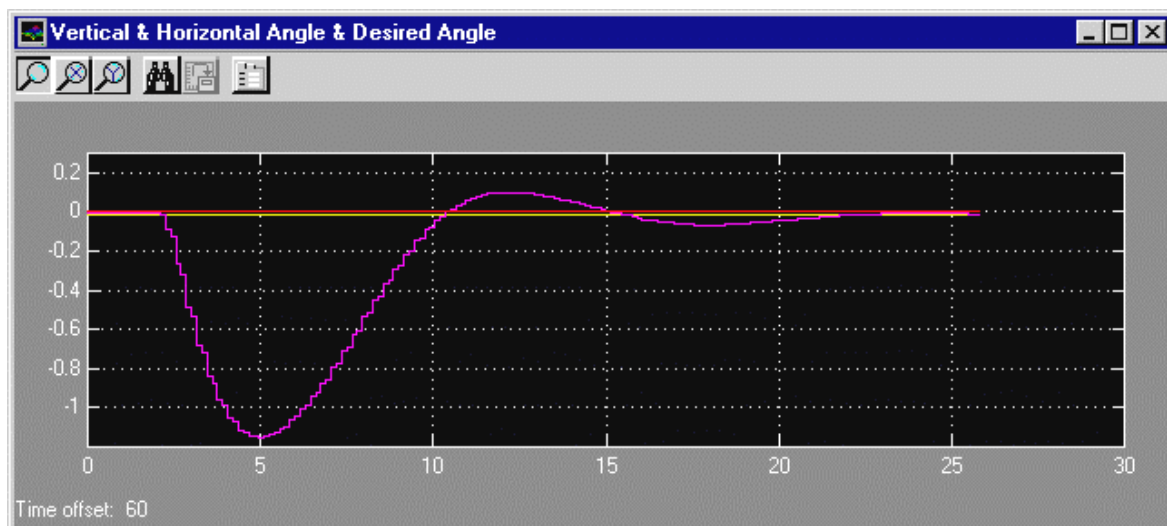
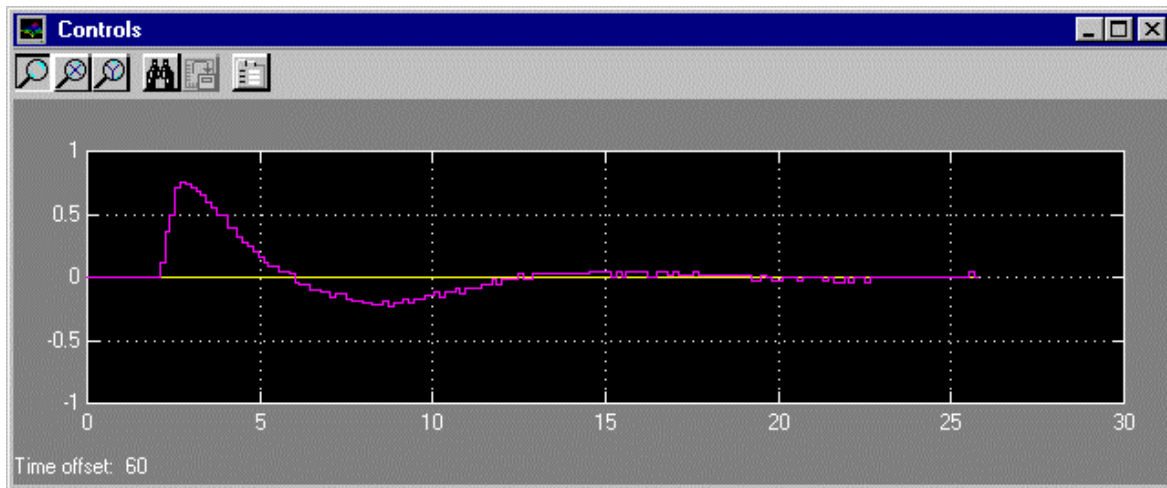


Figure 7-11: Real-time experiment results